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A STUDY OF VISUAL BINARY STARS

WITH REGARD TO THE ERRORS OF OBSERVATION
AND THE PROBLEM OF ORBIT DETERMINATION

BY

BENGT EKENBERG

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ACADEMICAL DISSERTATION

BY

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CARL BLOMS BOKTRYCKERI

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ORRIN J. LUNDGREN

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NO. 10

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Preface

Owing to the appearance of several new and important problems in astronomy, the study of the visual binary stars was no doubt disregarded to some degree during the first decades of this century. Since R. G. AITKEN published his *New General Catalogue of Double Stars* in 1932, the interest in these objects has considerably increased. This holds true especially of the problem of orbit determination.

The task of determining the orbit of a visual binary from the observations would be quite simple if the errors of measurement were negligible. Such is, however, by far not the case. On the contrary, the measurements are, especially in close pairs, of a considerable size, and this makes the orbit determination problem more difficult. The present paper has been planned with a view to this fact. Accordingly, the errors of observation are studied in the first two chapters, while the rest of the paper deals more directly with the problem of orbit determination.

The subject of the present investigation was proposed to me by Professor KNUT LUNDMARK, Director of the Lund Observatory. Professor LUNDMARK has continuously taken a keen interest in my work and given me much valuable support in various ways. For this, and for many fruitful discussions and suggestions, I beg him to receive my sincerest thanks.

My best thanks are due to Professor WALTER GYLLENBERG, Dr. ERIK HOLMBERG, Dr. ANDERS REIZ and the other members of the staff of the Observatory, for good advice and friendly criticisms during the course of the work.

I should like to express my best thanks to Mrs. MAJA ABDON for her careful and interested work with improving the language of the present paper.

Most of the technical part of the work has been performed by assistants. My sister, Mrs. ELSA SETH, has drawn the ephemeris curves, which are

necessary for the method of orbit determination given in Chapter IV. Mr. GOTTFRID ARENGERD has *i.a.* computed nearly all of the tables given in the Appendix. Much of the computation work for determining the orbits in Chapter V has been carried out by Mr. NILS-ERIC LANDGRAFF, who has also assisted me in drawing several of the diagrams and preparing the paper for the press. For their valuable help I express my thanks to them, as well as to other assistants.

Finally I beg to tender my gratitude to Kungl. Fysiografiska Sällskapet i Lund for pecuniary support.

Lund Observatory, January 1945.

BENGT EKENBERG.

CHAPTER I

Methods of eliminating or determining the systematic errors in visual double star measurements

As is well known, the determination of visual double star orbits, which is so important for the solution of several problems in astronomy, is based mainly on the following two measured quantities:

1) The *position angle*, or the angle (measured at the brightest of the two stars) between the direction to the North Pole and the line joining the stars. Nowadays this angle is counted from 0° to 360° in the direction from the north through the east.

2) The *angular distance* between the stars.

Although other observing methods (photographic and interferometric) have also come into use during the last decades, most of the measurements are still made visually, with the aid of a filar micrometer attached to the telescope. The way of making such observations will not be described here, as it has been treated in detail in several papers, *e.g.* in R. G. AITKEN's monograph *The Binary Stars*.¹

The micrometer measurements of double stars, as well as other kinds of measurements, are affected with both accidental and systematic errors. The usual method of eliminating the errors by repeating the observations can here be used advantageously only to a rather small extent, owing to the unfavourable influence of the systematic errors. The fact is that the systematic errors in micrometer measurements of double stars are — at least for the closer pairs — of such a size as to change the whole character of the problem of orbit determination. This has already been realized by the early workers in this field. Thus, TH. N. THIELE,² speaking of the degree of accuracy of an orbit determination, makes the following striking

¹ R. G. AITKEN, *The Binary Stars*. New York and London. 2nd Ed. (1935). Chapter III.

² TH. N. THIELE, *Undersøgelse af Omløbsbevægelsen i Dobbelstjernesystemet Gamma Virginis*. Diss. Copenhagen (1866). P. 91.

remark: »The existence of systematic errors deprives the problem of its exact character and makes reasonableness and judgement replace probability and calculation».

The aim of this chapter is to make a short survey of the possible ways of eliminating the systematic errors. This may be done directly in such a way that the errors do not appear in the results of the measurements. On the other hand, the said errors may be determined afterwards, and corrections may be applied to the measured values.

No method has been given for a direct elimination of systematic errors in the distance measurements, but in the case of position angles there exists the well-known method proposed by P. SALET and J. BOSLER in 1908.³ A small reversing prism is placed between the eye-piece and the observer's eye with its edges approximately parallel to the line joining the stars and its hypotenuse face parallel to the optical axis. The prism will thus invert the field of view, but the direction of the line through the stars is (approximately) unchanged. If an observer commits an error of a certain sign and size without the prism, he is likely to do the same when using the prism, but the latter error will act in the opposite way, so that the mean of the two readings should — theoretically — be free from systematic errors. At the same time, the differences between the results obtained without prism and these mean values will afford material for the investigation of the observer's personal errors. Generally four settings — two with and two without prism — are made for each star during a night's work.

A very extensive work for testing this method has been made by H. STRUVE,⁴ who has strongly recommended it. It has since then been used by several observers, although it has not been generally adopted. The references to observers using the method will not be given here; for detailed information the reader is referred to the comprehensive paper by P. SALET.⁵ The principal points of this paper may be summarized as follows.

Salet emphasizes that the observations with and without prism must be made immediately after each other. Some observers have contented themselves with making the observations with the prism at a later date, or even effecting them only occasionally in order to determine their systematic errors. However, this practice cannot be recommended and the reason is that the systematic errors may depend on time, as well as on several other circumstances, *e.g.* the distance between the components and their magnitude difference, the quality of the images, the transparency of the sky, and the

³ BA 25, p. 18 (1908).

⁴ Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, 1911, pt. 1, p. 41.

⁵ JO 11, p. 1 (1928).

disposition of the observer. The only way of eliminating the influence of all these circumstances is to carry out the two kinds of observations at the same time.

H. Struve, as well as many other observers, used to incline his head when observing, so as to get the line through the stars either parallel or perpendicular to the line joining the eyes. The idea is that small differences in inclination should very easily be detected in these positions. The systematic errors are also supposed to be smaller than usual in these two positions. However, there is no special reason why it should be so. (It may also be pointed out that a possible astigmatism in the observer's eye will modify the conditions, and that such an effect may be present even in the horizontal or the vertical direction.) It is true that most observers have the feeling that it is easier to measure the angles in one of the two positions quoted. But Salet shows from a comparison of Struve's results that the inclination of the head to either side may produce serious systematic errors, as there is a clear difference in the results if the head is inclined to the right or to the left. Salet therefore argues that the observer should always keep his head in its natural position. — It seems, however, that this advice has not been generally followed.

The above considerations show that it would be of advantage to observe either horizontally or vertically without being compelled to incline one's head. This may be done by using the reflecting prism in a way different from that used by Salet and Bosler. DAWES⁶ has made use of this device already in 1833, and since then it has been employed by several observers. To eliminate the systematic errors as far as possible, the observations are often made in all of the four possible positions: companion to the left, companion to the right, companion above and below the primary. For the reasons quoted above, this method must, however, generally be considered inferior to that of Salet and Bosler.

It seems judicious to combine the two methods by using two prisms, one working as in Dawes' method and one as in that of Salet and Bosler. This has been done by K. SCHILLER.⁷ As Salet points out, this method involves the disadvantage that the two prisms may hinder the eye from occupying the correct position in front of the eye-piece. The loss of light, which must naturally be considered even when only one prism is used, is another drawback.

The first of these difficulties may be overcome by the use of an apparatus devised by M. HAMY.⁸ In this case, the reflecting prism (or the two prisms)

⁶ Mem RAS 8, p. 62 (1833).

⁷ AN 192, col. 1 (1912).

⁸ CR 185, p. 978 (1927) = BA Mémoires et Variétés 5, p. 375 (1928) = Revue d'Optique 1928, p. 37.

is placed between two small objectives, and this system is inserted in the telescope; this arrangement enables the observer to hold his eye in the usual position in front of the eye-piece.

Salet finally suggests that the method proposed by himself and Bosler should be adopted by the International Astronomical Union; this has also been realized in 1932.⁹

N. V. KOMENDANTOFF¹⁰ has made extensive measurements of double stars, using a reflecting prism in such a way that the positions of the companion with and without prism are symmetrical to the vertical. As SALET¹¹ has pointed out, this way of using the prism is not the one which is indicated by himself and Bosler, and there is no guarantee that the systematic errors are really eliminated in measurements of this kind.

An extensive study on systematic errors in double star measurements has been published by G. V. SIMONOW,¹² who *i.a.* has made an investigation of the method of Salet and Bosler. Simonow points out a difference between the way of observing without and with the prism: in the first case the »fixed» stars are bisected by the »movable» micrometer threads, in the second case the stars are »movable» and are made to rotate twice as fast as the threads. Simonow means that this difference makes it improbable that the errors with and without the prism will be of the same size. His results do not speak in favour of the method, and he seems rather pessimistic about it. It should, however, be pointed out that it is not necessary to reject the method because its effect is greater for some observers than for others.¹³

When the systematic errors are not directly eliminated, it is desirable to determine them, if possible, in order to be able to apply suitable corrections to the observed values. Unfortunately, this task is rather difficult, and the results obtained are, on the whole, far from satisfactory. This is partly due to the fact that a number of different arguments must be considered, such as those already mentioned.¹⁴ Further arguments are the inclination of the line joining the stars, the altitude of the stars, the instrument, *etc.* The effects of these different arguments are interwoven with each other. Some of the arguments are not apt to be mathematically expressed. The existence of accidental errors, which are generally rather large, represents another difficulty. Even though it is hardly ever possible make a complete determination of the systematic errors, this fact is, however, no reason to desist from

⁹ Trans IAU Vol 4, 1932, p. 157.

¹⁰ Pulk Publ Ser 2, Vol 47 (1935).

¹¹ JO 19, p. 14 (1936).

¹² Berl-Bab Veröff 11:5 (1937). (Diss.)

¹³ Cf. the recension in Observatory 61, p. 29 (1938).

¹⁴ See p. 8—9.

studying them, all the more as the knowledge acquired by such a study may quite possibly be of value in other branches of science.

Thus it is not surprising that a great number of investigations have been devoted to this subject. The investigations have mainly dealt with the distance measurements, which is quite natural since these are more likely to be affected with systematic errors than the measurements of position angles. The following summary of the various kinds of methods for the determination of systematic errors will thus principally relate to the distances. However, most of the methods may also be applied to the position angles.

The most important methods of investigating the systematic errors may be classified as follows:

- 1) Comparison with artificial stars.
- 2) Comparison with measurements by other visual observers.
 - a) Direct comparison with mean values.
 - b) Comparison with computed orbits.
- 3) Comparison between values for close pairs, as measured with large instruments, and wide pairs, as measured with small instruments.
- 4) Comparison with values obtained by other measuring methods.
 - a) Comparison with photographic measurements.
 - b) Comparison with interferometer measurements.

1) *Comparison with artificial stars.* W. STRUVE¹⁵ seems to have been the first to use artificial stars for the purpose of correcting his measurements, but the most important work in this field was, as is well known, performed by his son, O. STRUVE.¹⁶ The latter carried out three extensive series of measurements on artificial stars: the first in the years 1853, 55 and 56, the second in 1866, and the third in 1876. The artificial stars consisted of small ivory cylinders (with diameters of 0.34, 0.23 or 0.10 inches), which could be placed into holes in a black iron disk. The disk was circular with a diameter of 35 inches and was erected at a distance of 1.7 miles from the telescope. The instrument was the famous 15-inch refractor of the Pulkowa Observatory, the same instrument that was used by Struve for his measurements of real double stars during half a century (1839—89). The angular distance between the artificial stars as seen from the telescope could be varied from about 0".5 to about 26", the number of possible distances being 15. The direction of the line joining the artificial stars could also be changed by turning the disk in its own plane (about 30° or 15° each time). If the stars are identical, the errors will obviously be the same for any two directions differing by 180°. From a comparatively small number of measurements Struve concluded that the same was true even if the components were

¹⁵ *Mensurae Micrometricae*. St Petersburg (1837). P. 142.

¹⁶ *Pulk Obs* 9 (1878); *Suppl to* 9 (1879); and 10 (1893).

of unequal sizes. Altogether some 1250 direction measurements and 600 distance measurements were made. As a direction measurement generally consisted of three settings, and a distance measurement of two settings of double distance, the total numbers of settings were no less than approximately 3750 and 1200.

By comparing the measured values with the correct ones, which could be determined with a sufficient degree of accuracy from direct measurements on the disk, Struve could determine the corrections in each case. Treating these corrections according to the method of least squares, he derived formulæ giving the corrections as functions of the visual angle (*i.e.* the distance reduced to an average magnification) and the angle of inclination of the stars. The following formula, intending to give the corrections of the distances during the time 1853—78, will give an idea of the form of the expressions.¹⁷

$$(1) \quad \text{Corr.} = + \frac{0''.050 (g - 2.0)}{1 + 0.09 (4.2 - g)^2} + \frac{0''.15 \cos (2 \varphi' - 28^\circ.4)}{1 + 0.06 (5.2 - g)^2}$$

Here g is the visual angle and φ' the true angle between the vertical line and the direction of the line between the stars.

Struve considered the question whether his systematic errors were dependent on time, and for this purpose compared his results for real stars during the following three periods: 1839—43 (indicated as OΣ 1841), 1843—53 (OΣ 1848), and 1853—78 (OΣ 1865), the last of these being subdivided at 1865. He found that there existed considerable differences, both in the directions and in the distances, between the values obtained during the three periods.

Struve's work naturally aroused a very great interest among astronomers, and it may be mentioned that N. C. DUNÉR¹⁸ made observations of artificial stars analogous to Struve's but not so extensive. The disk with the artificial stars was placed on the tower of the church of Uppåkra, 2.5 miles from the Observatory. Dunér, however, doubted that the personal errors which are derived from the measurements of artificial stars are valid for real double stars.

Several objections were soon raised against the methods employed by Struve and also to some of the conclusions he made. A thorough criticism was delivered by THIELE,¹⁹ who *i.a.* pointed out that the above-mentioned assumption of the identity of opposite directions was not sufficiently supported by the observations. This assumption was founded on only one »cycle» (series of measurements for different values of the inclination angle) for directions only; no measurements of this kind were made for the

¹⁷ *Loc. cit.*, p. (100).

¹⁸ N. C. DUNÉR, *Mesures micrométriques d'étoiles doubles*. Lund (1876).

¹⁹ VJS 15, p. 314 (1880).

distances. Furthermore, according to Thiele, the results of this »cycle» rather indicated that two opposite directions are not equivalent when the stars are unequal. — Thiele also criticised the division into periods and claimed that this division had caused Struve to overlook changes which are present in his systematic errors.

Other weaknesses in the measurements of artificial stars as performed by Struve were pointed out by G. BIGOURDAN,²⁰ E. GROSSMANN²¹ and others. Thus, it seems to be of importance that the telescope was always directed horizontally when the artificial stars were measured, whereas the measurements of real stars were naturally carried out at higher altitudes. The different positions which the observer must occupy are likely to influence the results. — A serious objection to the method seems to be that the definition of the artificial stars was rather different from that of the real ones. Furthermore, it is possible that the colours of the stars may be of importance. For the third, and above all, the images of the real stars are not always steady. The latter fact has been expressed by T. LEWIS²² in the following often cited statement: »There is much the same difference between observing an artificial and an actual star as there is between fencing with a dummy and a living enemy; . . .»; and SALET,²³ perhaps still more strikingly, characterizes the images of the real stars thus: »On peut comparer ces dernières à l'œf qui se balance sur un jet d'eau, dans les tirs, et qui est bien plus difficile à viser qu'un carton fixe.»

Attempts to overcome the disadvantages just mentioned were made by BIGOURDAN,²⁴ among others. In the apparatus designed by Bigourdan the artificial stars consisted of holes in an opaque screen illuminated from behind. The real directions and the distances of the stars could be measured on the screen. A small objective produced a real image diminished to about $1/100$, and this image was measured with a micrometer. The whole apparatus was mounted in the same tube, so that the altitude could be changed at will. The light and the colours of the artificial stars could be varied with the aid of glass wedges, and the atmospherical disturbances were imitated by aid of gas burning above the light rays.

In the previously mentioned work by G. V. SIMONOW²⁵ the artificial star method was thoroughly investigated. In the apparatus used by Simonow

²⁰ Annales de l'Observatoire de Paris, Mémoires, 19 (1889), C.

²¹ E. GROSSMANN, Untersuchung über systematische Fehler bei Doppelsternbeobachtungen ausgeführt in Verbindung mit einer Bahnbestimmung des Doppelsterns » η coronae borealis». Diss. Göttingen (1892).

²² Observatory 31, p. 344 (1908).

²³ JO 11, p. 3 (1928).

²⁴ Loc. cit.

²⁵ Berl-Bab Veröff 11:5 (1937). (Diss.)

the artificial stars were obtained in the same way as in Bigourdan's arrangement. The screen was mounted at one end of a collimator, and from the collimator lens the light-rays passed through a small telescope having its eye-piece end turned towards the collimator. The whole apparatus was fastened in front of the objective of the 25 $\frac{1}{2}$ -inch telescope of the Berlin-Babelsberg Observatory. The investigation did not aim at the derivation of systematic corrections of measurements of real stars but only at the study of the systematic errors. However, Simonow arrived at the conclusion that it may also be possible to determine systematic corrections in this way, provided the measurements of the real and the artificial stars are carried out simultaneously, in order that the influence of a possible change in the observer's apprehension may be eliminated. A necessary condition is, however, that the true values of the direction and the distance of the artificial stars can be measured accurately.

For this purpose, Simonow suggested the use of the »comparison-image micrometer» constructed by F. J. HARGREAVES.²⁶ In this instrument, the images of two artificial stars are reflected into the field of view of the telescope, so that they can be directly compared with the stars to be measured. Simonow, however, proposed that the instrument should be used in such a way that the two pairs are measured alternately.

If Hargreaves' comparison-image micrometer is used in the way intended by its constructor, it makes the micrometer threads unnecessary. The two pairs are measured in such positions that the four stars are situated either along a straight line or at the corners of a parallelogram. As in all modern instruments of this kind, the magnitudes of the artificial stars are adjusted to match those of the real stars.

Hargreaves' micrometer was tried out on the 28-inch refractor of the Greenwich Observatory by C. R. DAVIDSON and L. S. T. SYMMS.²⁷ The two slit plates used by Hargreaves to obtain the artificial stars were replaced by a Wollaston prism (to produce the double star) combined with two Nicols (to regulate the brightness of the stars).²⁸

According to the authors, the use of the comparison-image micrometer results in measurements which are superior to those obtained with the filar micrometer, at least for close, or faint pairs.

Another kind of double-image micrometer has been designed by

²⁶ MN 92, p. 72 (1931), and MN 92, p. 453 (1932).

²⁷ MN 98, p. 176 (1938); see also Proceedings at Meeting of the Royal Astronomical Society, Observatory 61, p. 40—42 (1938).

²⁸ It ought to be remarked that the use of a double-refracting prism in this connection is far from new; the same idea has been developed in the very first paper read before the Royal Astronomical Society of London, in 1820, by the Rev. W. PEARSON. (Mem RAS 1:1, p. 67.)

M. DURUY.²⁹ The distances are measured by comparing the real pair seen with the right eye with an artificial pair seen with the left. The arrangement is such that the observer is looking downwards, as in a microscope. — It can, however, be objected that this binocular observing method may lead to systematic errors owing to differences between the eyes.

From the above discussion it is evident that the methods employed in the modern investigations are altogether different from Struve's. The object of the use of artificial stars is nowadays not mainly to determine systematic errors but rather to develop a new kind of observing instrument different from the filar micrometer. Although the recent investigations are thus not so closely related to the problem of the systematic errors, they have been mentioned here, since they seem to be of a very great interest. It is desirable that such methods should be further tried out with the aid of large instruments.

2) *Comparison with measurements by other visual observers.* a) *Direct comparison with mean values.* If it is assumed that the results of a certain observer, or the mean of the results of several observers, are free from systematic errors, then it would be possible to determine the absolute values of the systematic errors of other observers, provided the different sets of measurements are carried out at the same time. The last assumption is not necessary if the stars can be regarded as fixed with respect to each other, or if their relative position can be determined as a function of the time with a sufficient degree of accuracy. Such a determination may be brought out either in the form of an empirical formula or by the computation of an orbit. The last case will be treated below under b).

If the comparison values cannot be assumed to be free from systematic errors, they will of course only lead to the knowledge of the relative systematic errors, but even so comparisons of this kind may be of interest, and they may lead to a first approximation of the absolute systematic errors.

Thus is not surprising that several comparisons of this kind have been made. It is not intended to quote them all here, but it may be mentioned that O. STRUVE³⁰ carefully compared his own measurements with those of other notable observers, and that he collaborated with E. DEMBOWSKI and other observers, using a standard list of 30 double stars, chosen by Dembowski.³¹

²⁹ BSAF 51, p. 585 (1937); JO 21, p. 97 (1938); JO 23, pp. 20, 145 (1940).

³⁰ Pulk Obs 9 (1878), and 10 (1893).

³¹ See e.g. STRUVE's paper in VJS 11, p. 227 (1876); Annales de l'Observatoire de Paris, 1885, p. G 126; Pulk Obs 9, p. (139)—(145) (1878); Pulk Obs 10, Suppl (1893); and Misura Micrometrica di Stelle Doppie e Multiple fatte negli anni 1852—1878 dal Barone ERCOLE DEMBOWSKI, 2, p. XXXVI. Rome (1884).

An investigation of the systematic errors of all the more important observers of that time was made in 1908 by H. E. LAU³² whose material consisted of no less than 1006 slowly moving stars discovered by Wilhelm Struve. The number of annual means used for the comparison was about 13000.

Several other standard lists have been worked out in later years. Thus, S. DE GLASENAPP³³ has published a list of 28 pairs situated between 2^h and 3^h in right ascension. If all hours in right ascension are to be represented, as it is intended, a complete list of this kind would comprise no less than some 700 pairs, a number which is far too large in practice. This remark has been made by P. MULLER,³⁴ who also criticises the list from other points of view. Thus, Muller points out that Glasenapp's list contains some pairs which are difficult to observe, and he claims that only quite easy pairs should be considered. Against this statement it may be remarked that, although the most difficult pairs are of course not suitable since they give too large accidental errors, it is on the other hand not correct to restrict oneself to the easiest pairs, because such pairs will probably not give rise to the same systematic errors as those of average difficulty.

Another standard list, containing about 100 pairs, has been published by M. GIACOBINI.³⁵

The list proposed by Muller³⁶ consists of only 15 pairs whose separations range from 1".5 to 6" approximately. The object of the list is to furnish material for a study of the systematic errors of different observers in distance measurements as functions of the distance itself. While Glasenapp's and Giacobini's lists were mainly intended to be used for the investigation of the systematic errors of the earlier observers, Muller proposes that his pairs should be measured by all observers. He also gives provisional values of the distances which he has obtained with a double image micrometer designed by himself. According to Muller, this instrument³⁷ in which the double-refracting prism is movable not parallel to the axis of the telescope, but perpendicular to it, is far superior to the filar micrometer. He especially emphasizes that the number of revolutions made by the micrometer screw in the distance measurements is far greater than it is when a filar micrometer is used; thus errors in the screw will not be of any

³² AN 180, col. 17 (1909).

³³ CR 186, p. 681 (1928).

³⁴ JO 22, p. 181 (1939).

³⁵ Mesures d'Étoiles doubles faites à l'Observatoire de Paris, p. 12 (1934).

³⁶ *Loc. cit.*; BSAF 53, p. 323 (1939).

³⁷ Bull séances Soc Française de physique No 410 (1937) = Journal de physique et le Radium (7) 8 No 12 (1937); see also CR 205, p. 961 (1937); BSAF 51, p. 586 (1937); JO 21, p. 113 (1938); and BSAF 53, p. 323 (1939).

importance. — When it is considered that Muller's results are generally obtained with a refractor of only 6 inches aperture, it seems surprising that he wants them to be taken as standard values; but in any case the results obtained with his micrometer are so accurate that the instrument is no doubt worthy of being further tried out. — The circumstance that the number of stars chosen by Muller is small is unquestionably an advantage from the point of view that all observers are to use the whole list; but since the distance is not the only argument of importance for the systematic errors, it is clear that only an approximate knowledge of their dependence on the distance can be gained in this way. Further, as Muller himself points out, the investigation ought to be extended to smaller separations.

A question of fundamental importance for investigations of this kind has already been indicated. This question is, whether the systematic errors of different observers are so distributed that the mean result obtained by a sufficiently large number of observers (or what may be called a »mean observer») is free from systematic errors or not. Theoretical considerations can here at most give an indication whether such an assumption is plausible; the solution of the problem must be searched empirically. It may, however, be of interest to quote some statements on this important question, especially as opinions still seem to be divided.

In his paper on Castor, TH. N. THIELE³⁸ makes an extensive comparison between the values which he finds from interpolation curves and the results of different observers. On p. 6 in his paper, however, he makes the following statement: »Or, on n'a pas le droit de considérer la moyenne des plusieurs observations contemporaines comme s'approchant plus de la vérité que chaque observation individuelle, . . .» — Thus, the fact that Thiele used interpolation curves does not indicate that he expected to find the absolute values of the systematic corrections in this way. It seems, however, to have been understood so by F. C. HENROTEAU³⁹ who writes in the *Handbuch der Astrophysik*:

»An attempt was made to determine the personal error by means of the observations, assuming that, for a given epoch, the mean of the figures obtained by all the observers is correct, and comparing with this mean the measures of each observer.

This is the method followed by T. N. THIELE who chose Castor. . . .»

In an important essay on Double-Star Astronomy, T. LEWIS⁴⁰ writes: »From the very nature of the measures it is evident personality exists, but

³⁸ TH. N. THIELE, Castor, Calcul du mouvement relatif et critique des observations de cette étoile double. Festschrift i Anledning af Universitetets Firehundredaarsfest Juni 1879. Copenhagen (1879).

³⁹ Handbuch der Astrophysik, 6, Chapter 4, p. 316. Berlin (1928).

⁴⁰ Observatory 31, p. 345 (1908).

in an ever varying amount, not only between two observers but between the same observer on different occasions, and even in his measures of the same system. It is more or less to be considered as an accidental error.»

In his previously mentioned work on the method of Salet and Bosler, H. STRUVE⁴¹ makes the following statement: »... so haben die in den letzten Jahren von den HH. DOBERCK, LAU, LOHSE u.a. ausgeführten Untersuchungen an verschiedenen Doppelsternbahnen es sehr wahrscheinlich gemacht, dass sowohl in den Messungen der Positionswinkel wie auch der Distanzen systematische Fehler vorkommen, die bei der Mehrzahl der Beobachter in gleichem Sinne wirken und daher in den Mittelwerten nicht eliminiert werden, ...» This statement is approvingly cited by P. BAIZE in his paper on the measurement of double stars in BSAF,⁴² and in a paper on λ Ophiuchi⁴³ Baize makes the addition that this is the case especially in the vicinity of certain angles.

The last statement is supported by a result obtained by W. H. VAN DEN BOS.⁴⁴ In a discussion of the systematic errors in his measurements with the 10 $\frac{1}{2}$ -inch refractor of Leiden in the years 1920—25, he finds strong evidence that the apparent position angles (*i.e.* the angles reckoned from the direction straight downwards) 45° , 135° , 225° and 315° are underrepresented; and he continues: »This may be called a repulsive effect of the bisectrices of the quadrants, a systematic error which has been remarked by many double star observers.» If such an effect exists for the majority of the observers, it will naturally also be characteristic of a »mean observer».

In one of the foot-notes on p. 3 in his paper of 1928, SALET⁴⁵ makes the following remark to H. Struve's previously cited statement: »Remarquons à ce sujet que l'astigmatisme de l'œil n'existe pas indifféremment dans toutes les directions, ce qui permet de craindre pour certaines inclinations, une erreur systématique pour l'ensemble de tous les observateurs.»

DE GLASENAPP,⁴⁶ cited with approval by MRS. R. BONNET,⁴⁷ makes the following supposition: »... que les équations personnelles varient d'un astronome à l'autre comme des erreurs accidentelles».

⁴¹ Sitzungsberichte der Königlichen Preussischen Akademie der Wissenschaften, **1911**, pt. 1, p. 41—42.

⁴² BSAF **49**, p. 590 (1935).

⁴³ JO **24**, p. 125—126 (1941).

⁴⁴ Leiden Ann **14**:3, p. 13 (1925); see also »Micrometermetingen van dubbelsterren», Diss. Leiden (1925).

⁴⁵ JO **11** (1928).

⁴⁶ CR **186**, p. 681 (1928).

⁴⁷ Mesures d'étoiles doubles et notes relatives aux couples observés. (Observatoire de Paris.) Orléans (1938). P. 14.

On several occasions it has been noticed that an observer generally obtains larger distance values with a small instrument than with a large one, as long as the distance is smaller than a certain value depending on the separation power of the instruments. This fact will be further discussed in Chapter II.⁴⁸ Here it may be sufficient to remark that the mean of the values obtained by several observers will obviously be too large if the instruments used are not of a sufficient size.

If the systematic errors of different observers are closely studied, there will hardly be any doubt that a »mean observer» is not free from systematic errors, especially as regards the distance measurements. The same seems to be true regarding the position angles, although in this case the problem seems to be more difficult. In the next chapter these questions will be further discussed.

b) *Comparison with computed orbits.* The method of examining the systematic errors of an observer by comparisons with mean values obtained either directly or from interpolation formulæ or interpolation curves has the disadvantage that a comparison between the distances is possible only if these are equal or nearly equal. Why this is a disadvantage, is readily seen from the following fictive example. We assume that a »mean observer» measures all distances 10 % too large when the distance is about 0".5, but measures them correctly when the distance is about 1". In such a case the systematic error will not be detected by the above-mentioned method alone. A possibility of comparing the smaller distances to the larger is, however, given if the measured values can be compared to those obtained from an orbit computation. This is due to the fact that in a computed orbit the smaller and the larger distances are connected with each other in consequence of the law of areas and the condition of elliptical path. In this respect, a comparison with a well established orbit must thus be superior to direct comparisons, provided the range of distances in the orbit is not too small. A similar reasoning can be applied to the position angles. — The drawback of the method of using the orbits as comparison material is obvious: only about 1 % of the known double stars, or a little more than 200, have been subjected to orbit determinations, and many of these are not reliable enough to be used with advantage in this connection.

In several cases where an orbit has been computed, the computer has employed the residuals for an investigation of the systematic errors of the observers. Thiele's already mentioned papers on γ Virginis and on Castor were among the first of this kind. Without aiming at completeness, references to earlier and later papers of this type are given at the bottom of the

⁴⁸ See p. 63.

page.⁴⁹ Besides the references, the name of the double star and its number in ADS⁵⁰ or CPD⁵¹ are given in the foot-note.

Nearly all the systems included in the list are wellknown ones. Most of them have been recomputed later, and the results obtained from these orbits are thus not up to date. It may be mentioned that the orbits of *i.a.* η Cas, γ Vir and 44 i Boo have recently been carefully investigated by K. AA. STRAND,⁵² who has not, however, given the residuals of each observer separately. Especially Horst Simon's determination of the orbit of η Cas is criticised by Strand.⁵³

From the list it is also seen that W. Doberck has been most active in research of this kind. In 1908 he also published an investigation⁵⁴ which seems to be the only one in which residuals from several orbits have been used together in order to obtain the systematic errors of the principal observers. In that paper, in which he makes use of thirty orbits computed by himself, Doberck does not, however, take into consideration that an observer has often used different instruments, and that the size and quality of the instrument must influence the errors. The effect of the instrument is, however,

⁴⁹ TH. N. THIELE, *Undersøgelse af Omløbsbevægelsen i Dobbelstjernesystemet Gamma Virginis*. Copenhagen (1866).

TH. N. THIELE, *Castor, etc.* Copenhagen (1879).

W. DOBERCK, AN **141**, col. 153 (1896).

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» » » **173**, » **241** (1906).

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» » » **182**, » **27** (1909).

G. COMSTOCK, AJ **31**, p. 107 (1918).

W. RABE, AN **216**, col. 49 (1922).

W. S. FINSEN, UOC **68**, p. 343 (1926).

E. SILBERNAGEL, AN **233**, col. 145 (1928).

» » » » » **257** (1928).

» » » **234**, » **441** (1929).

H. SIMON, Königsberg **5** (1934).

G. V. SIMONOW, Berl-Bab Veröff **11:5** (1937). (Diss.)

γ Vir = ADS 8630.

Castor = ADS 6175.

η Cor = ADS 9617.

α Cen = CPD — 60° 5483.

γ Vir = ADS 8630.

ζ Cnc = ADS 6650.

ω Leo = ADS 7390.

H I 39 = Σ 3062 = ADS 61.

44 i Boo = ADS 9494.

Σ 1879 = ADS 9380.

Castor = ADS 6175.

α Cen = CPD — 60° 5483.

ζ Her = ADS 10157.

AC 7 = μ Her = ADS 10786.

η Cor = ADS 9617.

η Cas = ADS 671.

O Σ 285 = ADS 9378.

⁵⁰ New General Catalogue of Double Stars within 120° of the North Pole, by ROBERT GRANT AITKEN, in succession to the late ERIC DOOLITTLE. Carnegie Institution of Washington, Publication **417**, 2 Vols. (1932). This work will in the continuation be referred to as ADS.

⁵¹ Cape Photographic Durchmusterung for the Equinox 1875, by DAVID GILL and J. C. KAPTEYN. Annals of the Cape Observatory **3—5**, 3 Vols. (1896, 1897 and 1900). This work will in the continuation be referred to as CPD.

⁵² Leiden Ann **18:2** (1937).

⁵³ *Loc. cit.*, p. 66—67.

⁵⁴ AN **177**, col. 65 (1908).

examined by Doberck in another paper,⁵⁵ as regards W. Struve's, Dembow-ski's and his own measurements.

When the measurements obtained by an observer are published, they are often accompanied by comparisons with computed orbits. Among such comparisons may be mentioned that made by C. WIRTZ,⁵⁶ in which he, besides the comparisons performed in other ways, gives the residuals of his results from 50 orbits.

The data given in comparisons with orbits are generally: the dispersion in angle and distance and the mean of the errors in distance, taken for all the measurements together. Occasionally a rough division with respect to time or distance has been made.

In Chapter II of this paper the results obtained by some prominent observers will be compared with computed orbits, and some questions arising from the comparison will be discussed.

3) *Comparison between values for close pairs, as measured with large instruments, and wide pairs, as measured with small instruments.* This method has been proposed by P. SALET.⁵⁷ The principle is that the pair to be measured with the telescope is compared to a similar, wider pair, which is seen in the finder under about the same vision angle. The wider pair can, on the other hand, be measured in the larger telescope with a considerably higher (relative) degree of accuracy. It should thus be theoretically possible to measure the close pairs with nearly the same relative accuracy as the wider ones. — In practice there may, however, be some difficulty in finding a pair which has the same appearance in the finder as the pair to be measured has in the telescope, and this may explain why this theoretically interesting method does not seem to have come into practical use.

A similar idea has recently been applied by W. RABE⁵⁸ in his extensive measurements with the 10 $\frac{1}{2}$ -inch refractor of the Observatory of München. The comparison is here obtained by using different enlargements in the same telescope. Details of the method are not given, but according to Rabe the results are very promising.

4) *Comparison with values obtained by other measuring methods.*
a) *Comparison with photographic measurements.* It suggests itself to try to check micrometer measurements with the help of measurements obtained by other kinds of methods. If the distance between the components is sufficiently large, the photographic method affords material for such a

⁵⁵ AN 182, col. 107 (1909).

⁵⁶ Strassburg Ann 4:2, p. 177 (1912), and 5:B 1 (1923).

⁵⁷ BA Mémoires et Variétés 6, p. 399 (1930).

⁵⁸ München Veröff 2, Nr 1 (1939).

comparison. Since it is only during a comparatively short time that photography has been systematically used in double star measurement, the visual results have, however, been employed for the testing of the photographic ones rather than inversely.

Photographic measurement of double stars can be successfully used only if the separation of the stars exceeds a certain limit. The fact is that if the images of the stars are too close to each other, disturbing effects may give rise to serious errors, especially in distance measurements. Following the classification proposed by E. HERTZSPRUNG,⁵⁹ the errors in photographic measurements of double stars may be divided into the following three groups:

- A. Errors inherent in the photographic image,
- B. Errors of measurement,
- C. Personal errors of interpretation of the image.

As Hertzprung points out, a precise discrimination between the three kinds of errors cannot, however, be made.

The principal photographic effects which belong to group A and which may cause errors of systematic character are the following: the Kostinsky repulsion, the turbidity effect, and the gelatine effect. The first of these will cause an increase in the measured distances, the others a decrease. For a comparison between the visual and the photographic measurements it does not seem necessary to enter more closely upon these effects, which have been investigated by S. KOSTINSKY,⁶⁰ H. E. LAU,⁶¹ G. EBERHARD,⁶² F. E. ROSS,⁶³ G. SHAJN,⁶⁴ D. REUIJL,⁶⁵ P. LABITZKE⁶⁶ and many others. A very clear theoretical survey of the vicinity effects has been given by J. JUNKES.⁶⁷ What is important to know in this connection is especially the limit of separation below which the effects have to be considered, and also, if possible, how to correct for them if the separation is below this limit. The last problem is very complicated, and it is generally agreed that it is better to make the observations visually if the separation is so small that systematic errors are to be expected in photographic measurement. This makes the question of the

⁵⁹ BAN 9, p. 253 (1942).

⁶⁰ Pulk Mitt 1, p. 139 (1906); 2, p. 17 (1907).

⁶¹ AN 192, col. 179 (1912).

⁶² Phys Z 13, p. 288 (1912); Phot Korr 58, p. 15 (1922); Potsdam Publ 26, Nr 84, p. 45—55 (1926); Handbuch der Astrophysik, 2:2, Kapitel 5, p. 456—458. Berlin (1931).

⁶³ ApJ 53, p. 349 (1921); The Physics of the Developed Photographic Image. Monographs from the Research Laboratory of the Eastman Kodak Company, Nr 5. Rochester, N. Y. (1924).

⁶⁴ AN 230, col. 369 (1927).

⁶⁵ Photographic Measures of Close Double Stars. Diss. Utrecht (1931).

⁶⁶ AN 258, col. 225 (1936).

⁶⁷ Zeitschrift für wissenschaftliche Photographie, 36, Heft 10—11 (1937) = Specola Vaticana, Comunicazione No 3 (1937).

allowable minimum separation still more important, and several investigations have been devoted to it. Some of the obtained results are summarized by Labitzke.⁶⁸ The most important circumstance is no doubt the distance (in mm) between the images on the plate, and since this distance depends on the focal length, it is clear that this is here the most important of the instrumental constants. Of course, the allowable minimum separation is far from exactly defined, but it may be mentioned that values ranging from 0.10 mm to 0.18 mm on the plate have been given. If the focal length of the instrument is 5 m, this will correspond to an angular separation of about 4" and 7", respectively. — In order to increase the separation of the images on the plate, J. A. KAZANSKY⁶⁹ has employed an enlarging camera with a positive lens.

Even if the errors inherent in the photographic image are likely to be the most important ones, the errors belonging to groups B and C above must not be neglected. The fact is that personal errors have been found when the same plate has been measured by two or more observers. The personal errors have recently been subjected to an extensive study under the direction of Hertzprung.⁷⁰ A plate carrying 66 exposures of Antares was measured by 45 persons. The measured images of the brighter star were diffraction images obtained by aid of a coarse grating so as to make them about equal to the central image of the companion. The results strongly indicate the existence of personal errors, and measurements of this kind should therefore be carried out by several observers if the utmost reliability is to be attained.

As in the case of visual work, experiments have been made with photographic measurement of artificial double stars, especially by E. PRZYBYLLOK,⁷¹ D. REUJL⁷² and P. LABITZKE.⁷³ Labitzke is of opinion that laboratory experiments of this kind are not suitable for the investigation of personal errors, but that they may quite possibly be of value for the study of purely photographic effects.

Provided the separation of the stars is large enough, the systematic effects in photographic measurements seem to be of much less importance than in visual work. As has been emphasized by E. HERTZSPRUNG,⁷⁴ this is the main advantage of the photographic method, since a repetition of measurements will be of much greater value than in the case of visual work. Since the measurement of one double star image may be said to be equivalent

⁶⁸ *Loc. cit.*

⁶⁹ Russian Astronomical Journal **11**, p. 519 (1934).

⁷⁰ *Loc. cit.*

⁷¹ Königsberg Astr Beob **45**:4, p. 9—10.

⁷² *Loc. cit.*

⁷³ *Loc. cit.*

⁷⁴ PASP **49**, p. 309 (1937); BAN **9**, p. 311 (1942).

to an ordinary visual measurement, and since about 50 exposures of a double star may be made on the same plate, it follows, according to Hertzsprung, that photographic work is able to secure nearly a decimal more than visual determinations.

Comparing visual and photographic measurements, there remains the important question whether systematic differences exist even if the separation exceeds the previously discussed minimum distance for reliable photographic measurement. Further, if such differences really exist, it may be asked if the visual, the photographic, or perhaps both kinds of measurements are systematically in error. These questions, which are naturally to be considered principally in distance measurements, cannot be said to be settled as yet, but the results tend to show that if systematic differences exist, they are at least not very large.

No doubt there is good reason to expect that future photographic work will afford good means of testing the visual measurements. Of course, there is the disadvantage that no direct comparisons of this kind can be made for the smallest distances; but for pairs in orbital motion there still remains the possibility of an indirect comparison between the smaller distances measured visually and the larger ones measured photographically.

b) *Comparison with interferometer measurements.* Just as photography has turned out to be useful for the measurement of the wider double stars, the same applies to interferometric methods with respect to the closest pairs. For these pairs there is thus, theoretically at least, the possibility of examining the results obtained visually by comparison with those obtained with the interferometer. Although the interferometer measurements do not yet seem to be reliable enough to form the basis of such a comparison, it may be appropriate to discuss a few questions relating to them, the more as the interferometer method will probably be further developed in future.

The theory of the interference methods and their use for astronomical purposes was outlined already in the nineteenth century by A. A. MICHELSON,⁷⁵ M. HAMY^{75*} and others. An elaborate theory of the astronomical interferometer measurements has later been given by H. SPENCER JONES,⁷⁶ while the treatment recently given by R. H. WILSON, JR.⁷⁷ is restricted to the accuracy necessary from a practical point of view.

Among the more modern interferometer measurements of double stars may be mentioned those made by J. A. ANDERSON⁷⁸ on Capella with the

⁷⁵ Phil Mag 5:30, p. 1 (1890).

^{75*} BA 16, p. 257 (1899).

⁷⁶ MN 82, p. 513 (1922).

⁷⁷ Pennsylvania Publ, Astronomical Series, 6:4 (1941).

⁷⁸ ApJ 51, p. 263 (1920) = Mt Wilson Contr 9, p. 225 (1920).

100-inch reflector of the Mt. Wilson Observatory; by P. MERRILL⁷⁹ with the same instrument; by M. MAGGINI⁸⁰ with the 13-inch refractor at Catania; by W. H. VAN DEN BOS and W. S. FINSSEN⁸¹ with the 26 1/2-inch refractor of the Union Observatory, Johannesburg; and by R. H. WILSON, JR.,⁸² with the 18-inch refractor of the Flower Observatory, Pennsylvania. In these measurements the interference is produced by aid of two slits of fixed separation; interferometers built upon other principles have also been used, *i.a.* by A. DANJON.⁸³

The chief advantage of the interferometer method as compared to the visual method is no doubt that the former is practicable for much smaller distances than the latter.

The theoretical limit of separation for visual measurements is generally defined as the distance for which the central disk of the diffraction image of the companion falls upon the first dark diffraction ring of the primary. This distance in angular measure ($=\rho$) is obtained from the formula

$$(2) \quad \rho = 1.22 \frac{\lambda}{D}$$

where λ is the wave-length of the light and D the aperture of the telescope. If an average value of 5500 Å is assumed for the wave-length and D is expressed in inches, ρ will be given in seconds of arc by means of the formula:⁸⁴

$$(3) \quad \rho = \frac{5''.45}{D}$$

In practice the limit of separation will naturally depend not only on the aperture but on several other circumstances, such as the seeing, the magnitudes of the stars, *etc.* It is often possible to make fairly reliable measurements below this limit, and in Dawes' well-known empirical formula the constant in the right member is 4''.56.

According to Anderson,⁸⁵ the expression for the theoretical resolution limit in interferometer measurements is

$$(4) \quad \rho = 0.5 \frac{\lambda}{D}$$

⁷⁹ ApJ **56**, p. 40 (1922) = Mt Wilson Contr **11**, p. 203 (1922).

⁸⁰ Pubblicazioni del R Osservatorio Astrofisico di Catania (1925).

⁸¹ UOC **90**, p. 379 (1933) and **93**, p. 134 (1935).

⁸² *Loc. cit.*; see also Journal of the Franklin Institute, **221**, No 1, January, 1936 = Pennsylvania Reprint **34** (1936). (Diss.)

⁸³ Strasbourg Ann **3**:4, p. 181 (1936).

⁸⁴ In *The Binary Stars*, p. 57, R. G. AITKEN, apparently because of a computing error, gives the value 5''.03 of the constant in the right member.

⁸⁵ *Loc. cit.*

The resolving power should thus be more than twice that of the micrometer measurements with the same telescope. This is in good agreement with the results obtained by Wilson,⁸⁶ who states that pairs can be measured interferometrically down to 0".10 or 0".15 with the 18-inch refractor of the Flower Observatory, for which the resolution limit of micrometer measurements is theoretically 0".30. Since it seems to be possible to use the interferometer even below the theoretical limit, it appears that the resolving power is, in practice as well as in theory, a little more than doubled as compared to that of the micrometer.⁸⁷

The possibility of using the interferometer method is, however, restricted by the large loss of light, which causes that only relatively bright stars can be measured with instruments of a moderate size. Further, the magnitude difference between the components must not be too large.

As has been pointed out by W. H. VAN DEN BOS⁸⁸ and others, the reliability of the interferometer is often not so great as might be expected from the internal agreement of the results. This may be explained from the fact that atmospherical disturbances are liable to produce spurious effects. This explains why the interferometer measurements are probably, at least for the moment, not suitable as comparison material for the investigation of the systematic errors in micrometer measurement.

At last it may be mentioned that, in a few cases, the value of the major axis (in angular measure) obtained by the computation of a visual orbit founded on micrometer or photographic measurements, may be compared to the value which may be computed if the trigonometric parallax and the half-amplitude of the radial velocity curve of the system are known. This may be done by the following formula, which is easily derived:

$$(5) \quad a = p_{tr} \frac{P \sqrt{1 - e^2} \cdot K}{2\pi \sin i}$$

Here a , P , e and i are respectively the semi major axis, the period, the eccentricity and the inclination of the orbit; p_{tr} is the trigonometric parallax; K is the half-amplitude of the radial velocity curve; and π is the constant 3.14159 . . . The quantities are supposed to be measured in units which correspond to each other.

Such a comparison has been made in the case of α Centauri. The values

⁸⁶ *Loc. cit.*

⁸⁷ It may be mentioned that F. HENROTEAU (JO 23, p. 133 (1940).) has proposed a photoelectric method of measurement by which the theoretical resolution limit is still more reduced. Nothing, however, seems to have been published about practical experiments with this method.

⁸⁸ BAN 4, p. 103 (1927).

of P , e and i have been taken from the orbit by W. S. FINSSEN⁸⁹ ($P=80^y.089$; $e=0.5208$; $i=79^\circ.233$). The trigonometric parallax is, according to F. SCHLESINGER,⁹⁰ $0''.756$. The value of K in J. H. MOORE's *Fourth Catalogue of Spectroscopic Binary Stars*⁹¹ has been taken from H. SPENCER JONES⁹² and is: $K=3.90+6.03=9.93$ km/sec. If these values are inserted in the formula, the following value of a is found:

$$a=17''.55$$

This is in good agreement with Finsen's value, which is $a=17''.665$, and also with the corresponding value found from photographic measurements by E. HERTZSPRUNG,⁹³ which is $a=17''.584$.

It must, however, be admitted that the agreement is far better than that corresponding to the accuracy of the data, especially the determinations of radial velocity.⁹⁴ This comparison is thus not accurate enough to afford the means of an »absolute» determination of the visually measured distances, but it has been mentioned only because of its theoretical interest.

⁸⁹ UOC 68, p. 343 (1926).

⁹⁰ General Catalogue of Stellar Parallaxes. Yale University Observatory (1935).

⁹¹ Lick Bull 18, p. 1 (1936).

⁹² Cape Ann 10:8, p. 206 (1928).

⁹³ BAN 7, p. 329 (1936).

⁹⁴ Cf. e.g. the discussion by F. ZAGAR (Atti e Memorie della R Accademia di Padova 51, p. 3 (1935) = Padova Pubbl 46 (1935).) on the system of 48 Cassiopeiae.

CHAPTER II

Systematic and accidental errors in the measurements by different observers as obtained from computed orbits

Introduction.

In the previous chapter it was mentioned that orbit computers have often used their results for an examination of errors in the measurements carried out by different observers. However, a double star has generally not been so frequently measured by the same observer that the results from this star alone can afford a material large enough to permit reliable conclusions concerning the errors of the observer. Further, double stars differ from each other in several respects which may be of importance for the errors: distance, direction, magnitudes and colours and several other circumstances. In fact, there are no two pairs that are quite equal to each other. Thus, if an observer's errors are determined from only one double star, it must always be considered that the results cannot safely be applied to other stars.

From these points of view it seems to be better to combine the results obtained from several stars, although it must be considered that the material must necessarily become rather inhomogeneous in this way.

It must also be admitted that a material based on good orbits is naturally not to be considered as characteristic of all stars, since the best orbits are likely to be founded on observations that are better than the average. Therefore the errors found in this way must be expected to represent »lower limits» of the errors committed by the observer. Another fact acts in the same direction: it is sometimes necessary to exclude abnormally large errors, since these would otherwise vitiate the results.

The material.

The observers whose errors have been investigated are: O. Struve, W. Doberck, G. Comstock, R. G. Aitken, W. Bryant, G. van Biesbroeck, and

W. H. van den Bos. The results found for each observer will later be discussed separately.

In this investigation only such orbits have been used for which the residuals (observed value *minus* computed value) are given by the computer for each observer separately. By computing the ephemerides for other orbits it would have been possible to enlarge the material. This has, however, not been done, since it would have required very much work and since this investigation must in some respects be considered to be of a preliminary character.

The orbits have mostly been chosen according to W. S. FINSSEN's *Second Catalogue of Orbits of Visual Binary Stars*.¹ In this catalogue the orbits are divided into five classes: definitive, reliable, preliminary, parabolic, and indeterminate. Only orbits belonging to the first three classes have been used. Three orbits have been published too late to be included in Finsen's catalogue.

To avoid reiterations, references to all the orbits used are given in the following list. (Table 1.)

The columns of the list contain:

The number in BDS² or Inn.³

The number in ADS or CPD.

The name (or the names) of the system.

The class of the orbit according to FINSSEN's *Second Catalogue*.⁴ The letters signify: d=definitive; r=reliable; p=preliminary.

The name of the computer.

The source and the year of publication.

Altogether 42 orbits have been used, but some of them only for the special investigation of the errors of a certain observer.

The observations have always been taken together in annual means.

General remarks.

Before the results for each observer are given, some general remarks will be made.

The different arguments to be considered have generally been treated separately and independently of other arguments. This is, of course, not

¹ UOC **100**, p. 466 (1938).

² A General Catalogue of Double Stars within 121° of the North Pole, by S. W. BURNHAM. Carnegie Institution of Washington, Publication **5**, 2 Vols. (1906).

³ Reference Catalogue of Southern Double Stars. Compiled by R. T. A. INNES. Annals of the Cape Observatory **2:2** (1899).

⁴ *Loc. cit.*

Table 1. *References to orbits included in the comparison.*

BDS or Inn	ADS or CPD	Star	Class	Auth.	Source
335	520	Ceti 82 B = β 395	d	W. H. van den Bos	UOC 98, p. 342 (1937)
440	684	β 232	p	G. P. Kuiper	BAN 5, p. 231 (1930)
1144	1709	Andromedae 259 B = Σ 228	r	G. P. Kuiper	BAN 5, p. 231 (1930)
2187	3210	β 1185	r	G. P. Kuiper	ApJ 86, p. 166 (1937)
3876	5871	Σ 1037	—	G. Karmel	AN 269, p. 312 (1940)
4355	6483	O Σ 185	p	W. H. van den Bos	UOC 98, p. 347 (1937)
4771	6993	e Hya AB = Sp —	d	E. Slonim	Tashk Bull 5, p. 159 (1935)
5005	7284	Σ 3121	d	W. H. van den Bos	UOC 99, p. 448 (1938)
5103	7390	w Leo = Σ 1356	d	W. Dohereck	AN 173, col. 251 (1906)
5805	8189	O Σ 234	r	C. M. Riechert	AN 219, col. 227 (1923)
5811	8197	O Σ 235	d	R. G. Aitken	Lick Publ 12, p. 72 (1914)
5951	8337	β 794	d	W. S. Finsen	UOC 99, p. 454 (1938)
12461	— 48°4965	γ Cen = h 4539	d	W. H. van den Bos	UOC 96, p. 231 (1936)
6566	8974	25 CVn = Σ 1768	p	J. Jackson	Greenwich Cat, p. 213 (1921)
6578	8987	β 612	d	W. H. van den Bos	UOC 99, p. 455 (1938)
6641	9031	Σ 1785	r	W. Rabe	AN 231, col. 121 (1927)
6711	9094	β 1270	r	W. S. Finsen	UOC 99, p. 457 (1938)
6832	9229	Σ 1834	p	W. H. van den Bos	UOC 99, p. 458 (1938)
6842	9247	β 1111 BC	d	W. H. van den Bos	UOC 99, p. 460 (1938)
6955	9343	ζ Boo = Σ 1865	r	W. H. van den Bos	UOC 98, p. 349 (1937)
7001	9378	O Σ 285	d	G. V. Simonow	Berl-Bab Veröff 11:5, p. 47 (1937)
7251	9617	η CrB = Σ 1937	d	E. Silbernagel	AN 234, col. 441 (1929)
7360	9744	ι Ser = Hu 580	—	N. McLeod	AJ 47, p. 155 (1938)
7368	9757	γ CrB = Σ 1967	r	G. van Biesbroeck	Yerkes Publ 8:2, p. 24 (1936)
7416	9769	π^2 UMi = Σ 1989	p	R. H. Wilson, Jr.	AJ 44, p. 68 (1935)
7717	10157	ζ Her = Σ 2084	d	E. Silbernagel	AN 233, col. 145 (1928)
7929	— 34°6803	Scorpii 185 B = β 416 = M1bO 4 AB	d	J. Voûte	BAN 2, p. 181 (1924)
8038	10598	Σ 2173	d	R. G. Aitken	Lick Publ 12, p. 116 (1914)
8230	10871	A 235	p	W. S. Finsen	UOC 98, p. 353 (1937)
8353	11060	O Σ 341	p	R. G. Aitken	Lick Bull 11, p. 85 (1923)
8380	11111	73 Oph = Σ 2281	p	W. H. van den Bos	UOC 98, p. 354 (1937)
8933	11871	β 648	d	W. S. Finsen	AN 268, col. 229 (1939)
8965	11950	ζ Sgr = Hd 150	p	W. S. Finsen	UOC 98, p. 357 (1937)
9114	12145	Se 2 BC	—	H. N. Russell	Lick Publ 12, p. 138 (1914)
9602	12889	Σ 2576	r	K. Aa. Strand	BAN 8, p. 206 (1937)
9643	12973	ζ Sge AB = AGC 11	p	W. S. Finsen	UOC 98, p. 359 (1937)
10643	14499	e Equ AB = Σ 2737	d	W. H. van den Bos	UOC 90, p. 388 (1933)
11255	15176	24 Aqr = β 1212	d	W. S. Finsen	UOC 81, p. 112 (1929)
11222	15281	x Peg AB = β 989	r	W. J. Luyten	ApJ 79, p. 449 (1934)
11895	16173	Ho 296	p	W. H. van den Bos	UOC 90, p. 391 (1933)
12290	16665	β 80 AB	r	B. Tannenbaum	UOC 99, p. 463 (1938)
12404	16800	β 1266	r	S. Arend	AJ 45, p. 71 (1936)

quite correct from a theoretical point of view. It must be kept in mind that it has not been shown that the errors depending on different arguments are additive. Owing to the limited material this has, however, been the only way to proceed.

The following arguments have been considered:

1) *The different pairs.* The separate treatment of each pair is comparable to the investigations earlier referred to in Note 49 on page 20. The results may be expected to give some indication of the influence of the magnitude difference, which is probably the most important argument to be considered here; it should, however, be noticed that other arguments, such as the magnitudes themselves and the spectral classes may also play a rôle. The material is, however, hardly sufficient for a closer examination of the influences of the latter arguments. — By placing together the results for the different pairs it is sometimes possible to detect regularities in the errors.

2) *The time.* In the foregoing chapter it was mentioned that the errors of many observers no doubt vary with the time. If these variations are of a systematic character and, moreover, are slow, *i.e.* if they last for several years, it is to be expected that they can be revealed simply by plotting the residuals of a sufficiently large number of annual means as a function of the time. Of course, it must be presumed here that the systematic variations are large enough not to be concealed by the accidental errors. The method of comparing the mean values obtained for different periods often seems to be less suitable, since there is always the risk of choosing the division points inappropriately.⁵

If the errors of an observer are submitted to variations of a shorter duration, it will of course be much more difficult to detect them, and they are obviously not to be found from a study of the annual means.

3) *The angular distance between the stars.* Here the best method seems to be to plot the residuals against the distances, even if the method of dividing the material into distance groups also seems practicable, since the dependence of the residuals on the distances can be expected to be of a more regular character than in the case of the time.

The question whether the computed or the observed distance ought to be used as the independent variable is not so insignificant as might be expected. The fact is that for the smallest distances the errors are so large that the two alternatives may give rather different results. From a theoretical point of view, it must be considered more correct to choose the computed distances, since these must be expected to come much nearer to the truth than the observed distances. On the other hand, if the object is to use the

⁵ This was, according to Thiele, the case with Otto Struve; cf. p. 13.

results to derive corrections to observed distances, it is of course necessary to use these instead of the computed ones. Such corrections, however, are generally only of a statistical value, and in most cases it is not advisable to apply them rigorously to the observations. The dependence of the errors on the computed distances will thus generally be of more interest than the dependence on the observed distances.

Most of the well-determined orbits are characterized by short periods, and, in consequence of this, only close pairs have been taken into consideration. This explains why the dependence on the distances has been investigated only for distances smaller than about $1''.20$.

Instead of the angular distance, several investigators have used the visual angle under which the double star was seen in the instrument, as an argument.⁶ If the same instrument is used, the angular distance seems, however, to be of greater importance than the visual angle.

4) *The position angle.* As it has been stated in Chapter I, the inclination of the line joining the stars is likely to exercise influence on the errors. This inclination is equal to $\theta - q$, where θ is the position angle and q the parallactic angle. The latter can be computed if it is known under which hour angle the observation was made. Most observers, however, do not give the hour angles but only state that the observations have been effected near to the meridian. Under such circumstances, the value of q is nearly equal to zero, except when the double star is situated above the North Pole; in the latter case q is about 180° . In this paper, the following approximation has been applied. Except in the latter case — which is not very frequent — the position angle θ has been used as a substitute for the angle $\theta - q$; if the double star was situated above the North Pole, the position angle has been changed by 180° . — It must not, however, be neglected that some of the effects of the systematic errors may be blurred out in consequence of this approximation, so that only the more conspicuous effects can be expected to be revealed in this way.

The errors in the position angles can either be expressed in degrees or be reduced to arc by multiplying by the distance and dividing by 57.3. In this paper, the former way has been used, except for the comparison between the errors in distance and in position angle.

It ought to be added that the mean errors have been calculated from the mean deviations, since the larger errors might have received too large weights if the mean errors had been computed in the usual way.

The annual means have throughout been given the same weight.

⁶ Cf. p. 12.

Otto Struve.

It has already been mentioned that Otto Struve (* 1819, † 1905) made nearly all his observing work during the years 1839—89 with the 15-inch Pulkowa refractor, which was the largest refractor in the world at the time when it was erected. The results he obtained with this instrument are, as has already been mentioned, published in the Pulkowa Observations.⁷ — The theoretical resolution limit of the instrument⁸ is 0".36.

Struve's measurements of artificial stars have been discussed in Chapter I.⁹

The uncorrected values have principally been used, but in some cases comparisons have also been made with the values as corrected by OΣ.

Table 2 gives a summary of the stars used in the investigation and the data obtained for each star.

Table 2. *Otto Struve (uncorr.). Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\bar{\delta}_\theta$	σ_θ	$\bar{\delta}_\rho$ (in 0".01)	σ_ρ (in 0".01)
	m					
5871	0.0	5	-2.8 ± 0.9	2.1	+ 0.4 ± 2.0	4.4
7284	0.3	15,13	-2.4 ± 2.2	8.3	— 0.2 ± 1.4	5.2
7390	0.8	29,28	-4.0 ± 0.5	2.8	+ 5.8 ± 1.7	9.1
8189	0.4	8	+ 1.3 ± 1.4	3.8	— 6.9 ± 1.5	4.3
8974	3.5	3	-2.9 ± 0.9	1.6	— 8.3 ± 3.2	5.6
9229	0.1	4	-2.5 ± 0.5	0.9	+ 0.3 ± 4.6	9.2
9378	0.5	3	-8.7 ± 1.3	2.2	— 0.3 ± 3.6	6.1
9617	0.5	37	-2.7 ± 0.6	3.9	— 4.8 ± 1.3	8.0
10157	3.5	36,35	-3.6 ± 1.1	6.4	— 1.7 ± 1.8	10.9
11060	1.3	4	-0.8 ± 1.2	3.4	+ 4.5 ± 1.3	2.5
11111	1.5	4	-7.3 ± 1.1	2.3	+ 8.8 ± 5.0	10.0
12889	0.2	4	-5.1 ± 0.9	1.8	+ 3.8 ± 4.2	8.4
14499	0.5	5	-8.1 ± 0.9	1.9	+ 12.0 ± 2.9	6.5

The explanation of the symbols in this table (as well as in the analogous tables for other observers) is the following:

Δm = the magnitude difference between the components, according to H. N. RUSSELL and CH. E. MOORE: *The Masses of the Stars*.¹⁰

N = the number of annual means. If two numbers are given, the first applies to the position angles and the second to the distances.

$\bar{\delta}_\theta$ = the mean value of the residuals δ_θ (observed — computed) in the angles, and its mean error.

⁷ Pulk Obs 9 (1878); Suppl to 9 (1879); and 10 (1893).

⁸ Cf. p. 25.

⁹ See p. 11—13.

¹⁰ H. N. RUSSELL and CH. E. MOORE: *The Masses of the Stars*. Chicago (1940).

σ_θ = the dispersion in the values of the residuals in angle.

$\overline{\delta_\rho}$ and σ_ρ = the corresponding values for the distances, measured in *hundredths of seconds of arc*.

The predominance of negative values of $\overline{\delta_\theta}$ is very striking: 12 negative values against 1 positive, which is furthermore smaller than its mean error. Although one would instinctively agree to W. S. FINSSEN's¹¹ statement: »... it is difficult to conceive of a constant personal error in position-angle», it hardly seems possible to avoid the conclusion that such an error really exists in the case of Otto Struve. — The positive and the negative values of $\overline{\delta_\rho}$, on the contrary, are about equal in number.

The values found for ADS 10157 are not included in the following discussion; the errors are so large and irregular that they would have vitiated the results completely, and moreover this star deviates from the others because of its large value of Δm and also of the distances. — Similarly, a few single values of δ_θ have been rejected.

In the continuation, the position angles and the distances are to be treated separately.

Position angles. Fig 1 shows how the errors in position angle depend on the time. The excess of negative values is distinct, and it is further seen to be larger during the last decades than before 1853 or thereabout. — Table 3 gives time corrections derived from this material. (For the sake of simplicity, the corrections have been assumed to be constant during the periods when they nearly seemed to be so. Such approximations have also been used in similar cases.)

It may be of interest to compare these results with those obtained by Struve himself¹² from comparisons made on pairs for which no movement had been found. For the average visual angle $g=0''.76$, which nearly corresponds to the distances occurring in this paper, Struve finds the following differences between the three periods OΣ 1841, OΣ 1848 and OΣ 1865.¹³

OΣ 1841—OΣ 1865	$+3.90 \pm 0.69$	(30 stars)
OΣ 1848—OΣ 1865	$+1.58 \pm 0.39$	(48 »)
OΣ 1841—OΣ 1848	$+2.30 \pm 0.61$	(19 »)

From the present material the following mean values of the errors are found:

OΣ 1841	-0.1 ± 1.2	(8 values)
OΣ 1848	-2.3 ± 0.5	(38 »)
OΣ 1865	-4.4 ± 0.5	(53 »)

¹¹ UOC 68, p. 350 (1926).

¹² Pulk Obs 9, p. (115).

¹³ Cf. p. 12 of the present paper.

Table 3. *Otto Struve. Position angles. Corrections for different epochs (to be applied to the uncorrected values).*

Time	Correction	Time	Correction
1840.0	-0.6°	1863.5	$+4.0^{\circ}$
40.5	-0.2	64.0	4.3
41.0	$+0.3$	64.5	4.6
41.5	0.7	65.0	4.8
42.0	1.0	65.5	5.1
42.5	1.4	66.0	5.4
43.0	1.6	66.5	5.6
43.5	1.8	67.0	5.8
44.0—58.0	2.0	67.5	5.9
58.5	2.1	68.0—74.5	6.0
59.0	2.2	75.0	5.9
59.5	2.3	75.5	5.7
60.0	2.5	76.0	5.5
60.5	2.7	76.5	5.3
61.0	2.9	77.0	5.0
61.5	3.1	77.5	4.8
62.0	3.3	78.0	4.7
62.5	3.5	78.5—89.0	$+4.6$
63.0	$+3.8$		

The corresponding differences are:

$$\text{O}\Sigma\ 1841 - \text{O}\Sigma\ 1865 \quad +4.3 \pm 1.3$$

$$\text{O}\Sigma\ 1848 - \text{O}\Sigma\ 1865 \quad +2.1 \pm 0.7$$

$$\text{O}\Sigma\ 1841 - \text{O}\Sigma\ 1848 \quad +2.2 \pm 1.3$$

As is seen, the agreement between the different results is very good.

Struve also writes that his values during the first period do not need any constant correction, and his value of the constant term in his correction formula (or the error with opposite sign) for $\text{O}\Sigma\ 1865$ is $+4^{\circ}.66$. These statements agree well with the values $-0^{\circ}.1$ and $-4^{\circ}.4$ above.

On the other hand, Struve's conclusion¹⁴ that his systematic errors have not changed during the time 1853—78 is not confirmed. Struve finds the following difference between the results for $\text{O}\Sigma\ 1860$ (1853—65) and $\text{O}\Sigma\ 1870$ (1865—78):

$$\text{O}\Sigma\ 1860 - \text{O}\Sigma\ 1870 \quad -0^{\circ}.25 \pm 0^{\circ}.73 \quad (21 \text{ stars})$$

and concludes that no change has taken place in his manner of observing during the whole period 1853—78. If two discordant values are rejected from the material of the present paper, the following mean values are found:

$$\text{O}\Sigma\ 1860 \quad -3.1 \pm 0.8 \quad (22 \text{ values})$$

$$\text{O}\Sigma\ 1870 \quad -5.5 \pm 0.6 \quad (31 \text{ » })$$

¹⁴ *Loc. cit.*, p. (112)—(113).

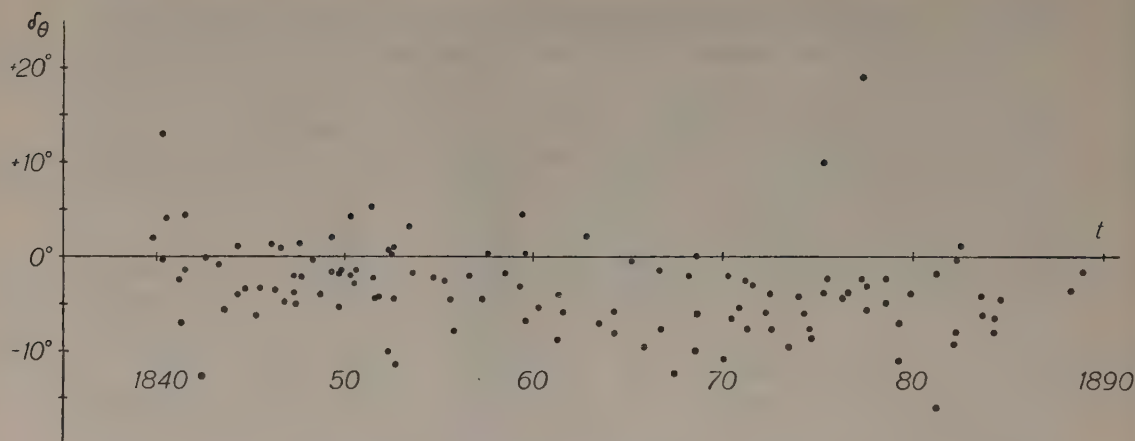


Fig. 1. Otto Struve (uncorr.). Dependence of the errors in position angle on the time.

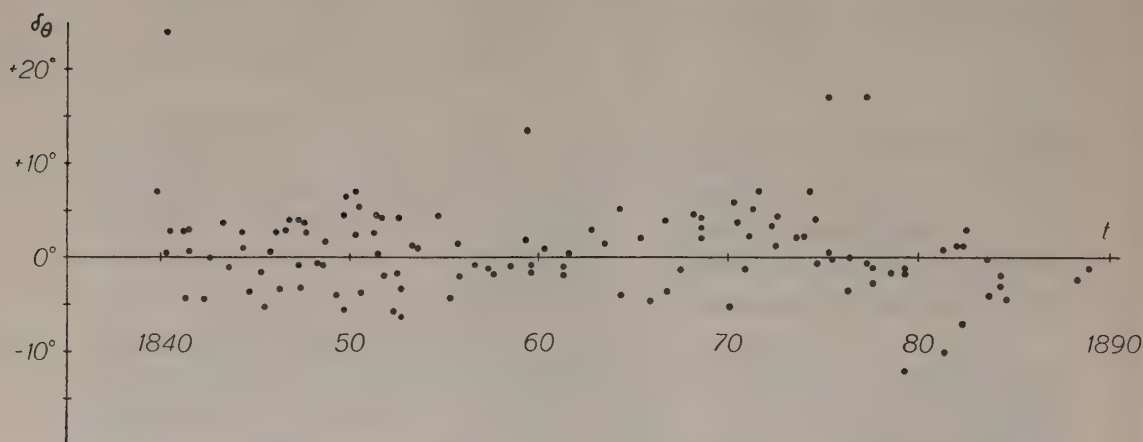


Fig. 2. Otto Struve (corr.). Dependence of the errors in position angle on the time.

and the difference is:

$$O\Sigma 1860 - O\Sigma 1870 \quad +2^\circ.4 \pm 1^\circ.0$$

It seems rather probable that this quantity is real.

According to E. GROSSMANN¹⁵ a successive change in Struve's apprehension of the distances took place about 1864, and a discontinuous change should have occurred between 1867 and 1868. To examine the correctness of these statements, the material has been augmented by the addition of the values of the distance errors between 1860 and 1870 for the following ADS

¹⁵ Untersuchung über systematische Fehler bei Doppelsternbeobachtungen . . . Diss. Göttingen (1892). P. 39.

stars: 8197, 9031, 9343, 9757 and 10598, or, together, 14 annual means. There is, however, no evidence of any change of the size assumed by Grossmann.

A plot has also been made of the values of the errors in position angle as corrected by Struve (Fig. 2). It shows that the individual values have, on the whole, been improved, and that the systematic run of the errors has been removed for the most part, as far as the dependence on time is concerned.

A study of the dependence of the residuals on the distance shows nothing remarkable, except for the smallest distances (below the resolution limit) for which the errors are large and irregularly distributed. Except for the smallest distances, the dispersion of the errors (in degrees) does not show any marked dependence on the distance.

To investigate the influence of the position angle on the errors, the material has been divided into eight groups according to the computed position angles, each group corresponding to 45° . The first sector contains the angles around 0° (337.5° — 22.5°), the second those around 45° , etc.

Since the results obtained from the original values appear somewhat doubtful, the values have been corrected for the influence of the time of observation by applying the corrections given in Table 3. The results obtained in this way for the different sectors are given in Table 4.

Table 4. *Otto Struve. Position angles. Errors for different position angle sectors. (Values corrected for time.)*

Sector	N	$(\bar{\delta}_\theta)_1$	$(\sigma_\theta)_1$
0°	16	-0.6 ± 0.6	2.4
45	22	-0.6 ± 0.8	3.8
90	17	$+0.8 \pm 0.7$	2.8
135	8	$+1.5 \pm 1.0$	2.8
180	15	$+1.4 \pm 1.0$	3.8
225	9	-0.2 ± 0.8	2.5
270	11	-3.2 ± 1.0	3.3
315	18	0.0 ± 0.5	2.2

Here $(\bar{\delta}_\theta)_1$ is the mean value of the errors in position angle corrected with respect to the time. $(\sigma_\theta)_1$ is the dispersion in the same values.

To judge from this table, the dependence on the position angles is not very pronounced.

It has previously been mentioned ¹⁶ that Struve assumed his errors for opposite directions to be identical, and that Thiele doubted the correctness of this statement. The present material does not support it either. If, in

¹⁶ See p. 11—13.

spite of this, the values from the opposite sectors are taken together and the results are compared with the corresponding values found by Struve,¹⁷ it is found that Struve's values are numerically larger, and that even the signs do not agree.

From the table it is further seen that the largest values of the dispersion are those for the sectors 45° , 180° and 270° , the smallest those for 0° , 225° and 315° . Thus, the opposite directions do not seem to be identical in this respect either.

There seems to be no doubt that Otto Struve's errors in position angle are systematic in an unusually high degree. In this case it even seems possible to improve the individual values by applying corrections to them. Such corrections may be tested by comparing the sum of the absolute values of the residuals before and after the application of the corrections. If the uncorrected values and the values corrected by Struve are compared in this way, it will be found that the sum of the residuals is diminished by 30 % by applying the corrections. In view of the considerable size of the accidental errors, this is a rather satisfactory result. If the corrections derived from the present material with respect to the time and the position angle are applied, the improvement is 43 %; and if only the time corrections are applied, the improvement is 38 %. Of course it is not surprising that these corrections give better results than Struve's own, since they are founded on the same material to which they are afterwards applied. In reality the two sets of corrections are probably of approximately the same value; and when the time corrections have been applied, the remaining corrections for different position angles are probably of little significance.

Distances. The dependence of the errors in distance on the time is shown in Fig. 3. It appears that Struve measured the distances too large during the first years and the last decades but too small during the time between these intervals. — The corrections read off from the mean curve and expressed in $0''.01$ (as will usually be done in the continuation) are given in Table 5.

As in the case of the position angles, the results have been compared with Struve's own.¹⁸ For $g=0''.76$, he finds the following results (expressed in $0''.01$):

$\text{O}\Sigma\ 1841-\text{O}\Sigma\ 1865$	$+0.3 \pm 1.0$	(38 stars)
$\text{O}\Sigma\ 1848-\text{O}\Sigma\ 1865$	-3.3 ± 0.5	(79 »)
$\text{O}\Sigma\ 1841-\text{O}\Sigma\ 1848$	$+6.5 \pm 1.0$	(23 »)

¹⁷ *Loc. cit.*, p. (90), the second and third terms of the right member of formula (23), with $g = 0''.7$.

¹⁸ Pulk Obs 9, p. (115).

Table 5. *Otto Struve. Distances. Corrections for different epochs
(to be applied to the uncorrected values).*

Time	Correction (in 0".01)	Time	Correction (in 0".01)
1840.0	— 8	1850.5—51.0	+ 7
40.5	7	51.5	6
41.0	6	52.0—52.5	5
41.5	5	53.0	4
42.0	4	53.5—54.5	3
42.5	3	55	2
43.0	2	56—58	+ 1
43.5	— 1	59—62	0
44.0	0	63—65	— 1
44.5	+ 1	66—68	2
45.0	2	69—71	3
45.5	3	72—74	4
46.0	4	75—78	5
46.5	5	79—81	6
47.0	6	82—84	7
47.5—48.0	7	85—88	8
48.5—50.0	+ 8	89	— 9

The present material gives the following mean values of the errors:

OΣ 1841	$+3.8 \pm 2.0$	(9 values)
OΣ 1848	-4.6 ± 1.3	(38 »)
OΣ 1865	$+1.2 \pm 1.0$	(67 »)

The corresponding differences are:

OΣ 1841—OΣ 1865	$+2.6 \pm 2.2$
OΣ 1848—OΣ 1865	-5.7 ± 1.6
OΣ 1841—OΣ 1848	$+8.3 \pm 2.3$

As is seen, the signs of the differences agree, but the numerical values show deviations. The same applies to the separate values for each period; Struve finds these values¹⁹ to be +3.3, —0.3 and +3.0 for the three periods 1841, 1848 and 1865, respectively.

While the foregoing results are in tolerable agreement, this cannot be said of the results for OΣ 1860 and OΣ 1870. For the difference between the values for these periods Struve finds:²⁰

OΣ 1860—OΣ 1870	$+3.0 \pm 1.8$	(29 stars)
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but from the present material is found:

OΣ 1860	-1.2 ± 1.3	(28 values)
OΣ 1870	$+2.9 \pm 1.4$	(39 »)

¹⁹ *Loc. cit.*, pp. (116) and (117), the values of κ with reversed signs.

²⁰ *Loc. cit.*, p. (112).

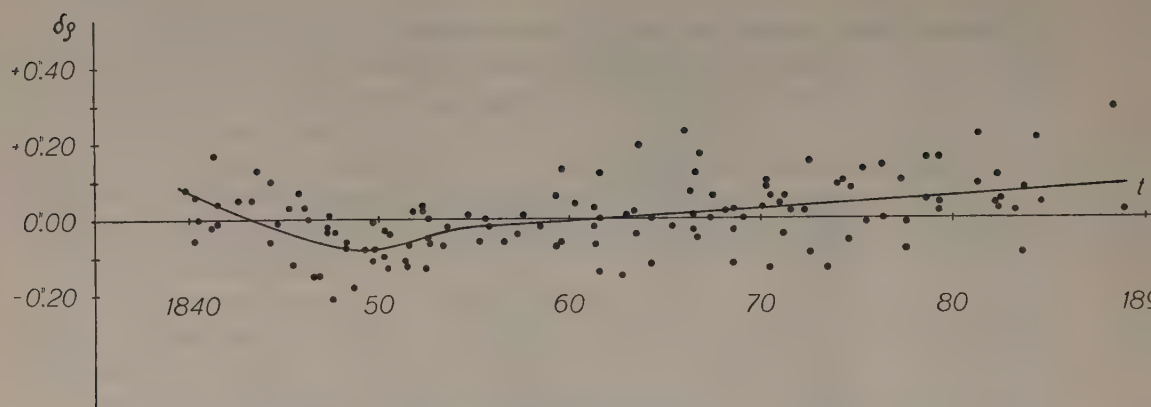


Fig. 3. Otto Struve (uncorr.). Dependence of the errors in distance on the time.

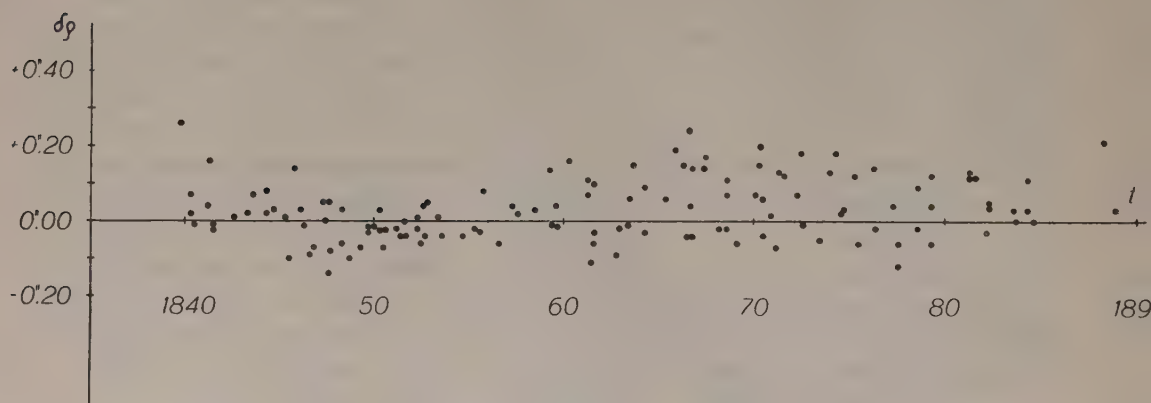


Fig. 4. Otto Struve (corr.). Dependence of the errors in distance on the time.

thus:

$$\text{O}\Sigma 1860 - \text{O}\Sigma 1870 \quad -4.1 \pm 1.9$$

Fig. 4 shows how the errors in Struve's corrected distances depend on the time. It confirms the often expressed opinion that Struve's corrections of his distances were less successful than those of the position angles. From the diagram it appears that the corrected distances are too large during the periods 1839—45 and 1855—89 and too small from 1845 to 1855.

The errors in distance for the uncorrected values have been plotted against the computed distances. No regular dependence can be traced, except for the very smallest distances ($\rho_c < 0''.35$), for which all the residuals are negative. As might be expected, the dispersion increases with the distance.

In the same way as in the case of position angles, the material has been divided into eight sectors according to the position angles. The distance

values have been corrected according to Table 5. The results are recorded in Table 6.

Table 6. *Otto Struve. Distances. Errors for different position angle sectors. (Values corrected for time.)*

Sector	N	$\overline{(\delta\rho)}_1$ (in 0''.01)	$(\sigma\rho)_1$ (in 0''.01)
0°	16	-3.2 ± 1.6	6.4
45	23	-3.6 ± 1.6	7.8
90	17	$+4.5 \pm 2.1$	8.7
135	8	-3.5 ± 2.3	6.6
180	15	-4.0 ± 1.3	5.0
225	10	-2.5 ± 1.6	5.0
270	11	$+8.4 \pm 1.8$	6.0
315	18	$+2.1 \pm 1.3$	5.7

In this case the agreement between the values for the opposite directions is much better than in the case of the position angles. Thus it may be of interest to study the results obtained by taking the opposite sectors together. The following mean values of the errors are found in this way:

Sectors	$\overline{(\delta\rho)}_1$ (in 0''.01)
0°, 180°	-3.6 ± 1.0
45, 225	-3.2 ± 1.2
90, 270	$+6.0 \pm 1.5$
135, 315	$+0.4 \pm 1.3$

There seems to be a strong indication that Struve measured the distances too large, especially if the direction was vertical, and too small, especially if it was horizontal. — The above values may be compared with those computed from Struve's own formula.²¹ His values (with reversed signs) are:

Angles	Errors (in 0''.01)
0°, 180°	-6.0
45, 225	-3.2
90, 270	+6.0
135, 315	+3.2

Considering that the methods are uncertain, and that the material is rather scarce for such an investigation, it must be stated that the agreement is a rather good one.

²¹ *Loc. cit.*, p. (100), the second term of the right member of formula (33''), with $g = 0''.7$; formula (1), p. 12 in this paper.

The run in the values of $(\sigma_\rho)_1$ is rather regular, but the values for the opposite directions do not agree.

A comparison between Struve's uncorrected and his corrected values shows that the sum of the absolute values of the residuals has been reduced by 10 %. The corrections derived from the present material with respect to time and position angles give an improvement of 24 %; the time corrections alone give an improvement of 13 %. Thus the improvement is less satisfactory than in the case of position angles. It seems, however, that the following combination of corrections may be practicable:

1) The corrections for different epochs, as derived from the present material (Table 5);

2) The corrections with respect to the inclination angles, as derived by Struve.²²

William Doberck.

The observations made by William Doberck (* 1852, † 1941) cover a time of no less than sixty years (1875—1934), and thus his measurements might be expected to furnish a very good material for the investigation of personal errors. Unfortunately, his observations were carried out with comparatively small instruments, some of which were, moreover, not of a satisfactory quality. The instrument he used before 1896 was the equatorial of the Markree Observatory, Ireland, with an aperture of about 13 inches (theoretical resolution limit 0".41). This instrument was mounted in the open air, as may be found from Doberck's interesting description.²³ In 1896 he took up double star work at the Hongkong Observatory, where he used a 6-inch equatorial (theor. resolution limit 0".91); this instrument was also mounted in the open air, and, according to Doberck,²⁴ it was most inferior. In 1908 he continued his observations at his private observatory at Kowloon, Sutton, England, using a 7 1/4-inch refractor (theor. resolution limit 0".75). He also made two shorter series of observations, at Copenhagen and at Columbus, Ohio.

The results of his measurements during the time 1875—1927 have been collected in the *Astronomische Nachrichten*;²⁵ references to his later measurements are given below.²⁶

The data for each star used in the investigation are given in Table 7.

²² *Loc. cit.*, p. (100), the second term of the right member of formula (33"); see also the table on p. (100)—(102).

²³ AN 92, col. 33 (1878).

²⁴ See AN 197, col. 90 (1914).

²⁵ AN, Erg.-Hefte Band 7 (1927).

²⁶ AN 239, col. 49 (1930); 243, col. 121 (1931); 248, col. 187 (1933); 254, col. 42 (1934).

Table 7. *Doberck. Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\bar{\delta}_\theta$	σ_θ	$\bar{\delta}_\rho$ (in 0''.01)	σ_ρ (in 0''.01)
	m					
1709	0.9	14	-1.4 ± 1.0	3.9	$+0.9 \pm 2.2$	8.2
5871	0.0	14, 12	$+0.1 \pm 2.2$	8.3	$+3.0 \pm 3.5$	12.3
7390	0.8	7, 5	-1.2 ± 2.1	5.6	$+1.8 \pm 2.3$	5.1
8189	0.4	3	-0.1 ± 2.1	3.7	$+5.7 \pm 7.9$	13.6
9378	0.5	8	-0.4 ± 1.6	4.6	$+6.1 \pm 2.4$	6.7
9617	0.5	21	$+0.8 \pm 0.5$	2.2	$+0.7 \pm 1.7$	7.6
10157	3.5	23, 21	$+0.2 \pm 0.3$	1.6	-2.1 ± 1.9	8.9
11111	1.5	9, 7	$+2.5 \pm 1.8$	5.4	$+8.6 \pm 6.4$	17.0
12889	0.2	11	$+0.4 \pm 0.3$	0.8	$+2.6 \pm 4.4$	14.7
14499	0.5	8, 6	$+2.2 \pm 0.8$	2.1	-1.2 ± 6.7	16.5

Like all the corresponding tables, this table is explained in the same way as Table 2 (p. 33).

The table does not show anything of particular interest as far as the angles are concerned, but in the values or $\bar{\delta}_\rho$ there is a marked excess of positive values. The dispersions are rather large, which, considering the instrument used, is of course not surprising.

Since the instrument is likely to be of great importance for the results, the material has been divided in the continuation according to the instruments. Owing to the small number of observations with each instrument before 1908, these observations have not been further investigated. Of the remaining measurements with the 7 $\frac{1}{4}$ -inch, those made on the star ADS 10157 were excluded.

It is found that the accidental errors are so large that conclusions regarding the systematic errors can hardly be drawn with certainty. From a study of the dependence of the errors in distance on the computed distances it appears, however, that Doberck measured the distances too large especially when they were below the theoretical resolution limit. The mean of the distance residuals for the 7 $\frac{1}{4}$ -inch refractor is (in 0''.01):

$$+3.6 \pm 1.5$$

This value is founded on 61 annual means, most of them for distances smaller than 1".

George C. Comstock.

The double star observations performed by George Comstock (* 1855, † 1934) were made with the 15 $\frac{1}{2}$ -inch refractor of the Washburn Observatory, Wisconsin, during the years 1887—1919. They are to be found in the

publications of the same observatory.²⁷ — The theoretical resolution limit of the instrument is $0''.35$.

A summary of the stars used and the data for each star is found in Table 8.

Table 8. *Comstock. Stars used in the comparison; results for each star.*

ADS or CPD	Δ_m	N	$\bar{\delta}_\theta$	σ_θ	$\bar{\delta}_\rho$ (in $0''.01$)	σ_ρ (in $0''.01$)
	m					
6993	1.5	5	-1.2 ± 2.6	5.8	-2.4 ± 1.5	3.4
7284	0.3	18	$+0.2 \pm 0.5$	2.0	-1.1 ± 1.7	7.0
7390	0.8	5	-0.5 ± 0.8	1.7	-8.0 ± 2.0	4.5
8189	0.4	12	-0.3 ± 0.4	1.4	$+2.4 \pm 1.4$	5.0
8987	0.0	14	$+2.6 \pm 1.2$	4.6	-0.4 ± 1.1	4.2
9378	0.5	17	-0.1 ± 0.8	3.1	-1.7 ± 0.9	3.6
9617	0.5	23	$+0.3 \pm 0.3$	1.5	-6.2 ± 1.1	5.3
10157	3.5	25	$+0.8 \pm 0.6$	2.8	$+7.8 \pm 2.2$	11.1
-34°6803	2.0	15	-2.4 ± 0.6	2.2	-29.9 ± 2.9	11.3
11871	3.5	14	$+0.7 \pm 1.0$	3.6	$+6.1 \pm 2.2$	8.1
11950	1.0	3	-10.9 ± 6.4	11.1	-3.3 ± 4.0	7.0
15176	0.1	5	-0.9 ± 1.4	3.0	$+0.6 \pm 0.6$	1.4
16665	1.0	5	-3.9 ± 2.7	5.9	-7.6 ± 1.4	3.1

In the table there is an excess of negative values of $\bar{\delta}_\rho$. — The three double stars ADS 10157, CPD -34°6803 and ADS 11871 exhibit large errors in distance, which may be explained from the large magnitude differences of these stars, the second one being, moreover, situated far to the south. These stars have been excluded in the continuation.

Position angles. The measurements of position angles give no evidence of a change with the time.

No systematic dependence of the values on the distance has been found. The dispersion decreases rapidly when the distance increases.

To investigate the dependence on the position angles, a division into eight sectors has been performed in the same way as in the case of Otto Struve. The results for the different sectors are given in Table 9.

In the table $\bar{\delta}_\theta$ denotes the mean value and σ_θ the dispersion of the errors in position angle.

The values of $\bar{\delta}_\theta$ are small and do not show any regularity. On the other hand, the values of the dispersions show a regular run that may hardly be due to chance. The maximum dispersion is found for 225° , while the minimum values are those for the opposite direction, 45° , and for 0° . It may be noted again that there is no agreement between the results for the opposite directions.

²⁷ Washburn Publ 6:2 (1890), 10:1 (1896), 10:3 (1906), 10:4 (1921).

Table 9. *Comstock. Position angles. Errors for different position angle sectors.*

Sector	N	$\bar{\delta}_\theta$	σ_θ
0°	16	$+0.2 \pm 0.4$	1.5
45	16	$+0.4 \pm 0.4$	1.5
90	16	0.0 ± 0.6	2.3
135	15	-0.7 ± 0.7	2.7
180	9	$+0.1 \pm 1.1$	3.4
225	25	-0.9 ± 1.3	6.4
270	5	-2.2 ± 1.7	3.6
315	5	-0.2 ± 1.0	2.3

Distances. There is some indication that the errors in distance may possibly depend on the time, but the variations are irregular and not safe.

In Fig. 5 it is illustrated how the errors in distance depend on the computed distance. For distances smaller than $0''.45$ the systematic errors do not deviate much from the zero line, but for larger distances the predominance of negative values is very striking. Owing to the comparatively large size of the accidental errors, it seems that the mean curve is represented with sufficient accuracy by the two straight lines in the figure. If the corrections corresponding to these lines are applied, it will be found that the sum of the absolute values of the residuals is reduced by 17 %. — The conditions if the observed distance is chosen as abscissa are shown in Fig. 6.

The values of the errors have been corrected for the dependence on the distance (by means of the straight lines in Fig. 5), and the results for different sectors have been computed as before. See Table 10.

Table 10. *Comstock. Distances. Errors for different position angle sectors. (Values corrected for distance.)*

Sector	N	$(\bar{\delta}_\rho)_2$ (in $0''.01$)	$(\sigma_\rho)_2$ (in $0''.01$)
0°	16	-0.9 ± 0.9	3.7
45	16	-0.6 ± 2.1	8.4
90	16	-2.6 ± 0.9	3.5
135	15	-0.4 ± 1.2	4.8
180	9	$+2.6 \pm 1.2$	3.7
225	25	-1.8 ± 1.0	5.0
270	5	0.0 ± 2.2	5.0
315	5	0.0 ± 0.7	1.5

The mean value of the errors in distance corrected with respect to the distance is here denoted by $(\bar{\delta}_\rho)_2$, and $(\sigma_\rho)_2$ is the corresponding dispersion.

There is still an excess of negative values of the errors; this is due to the fact that the corrections have been made a little smaller than those

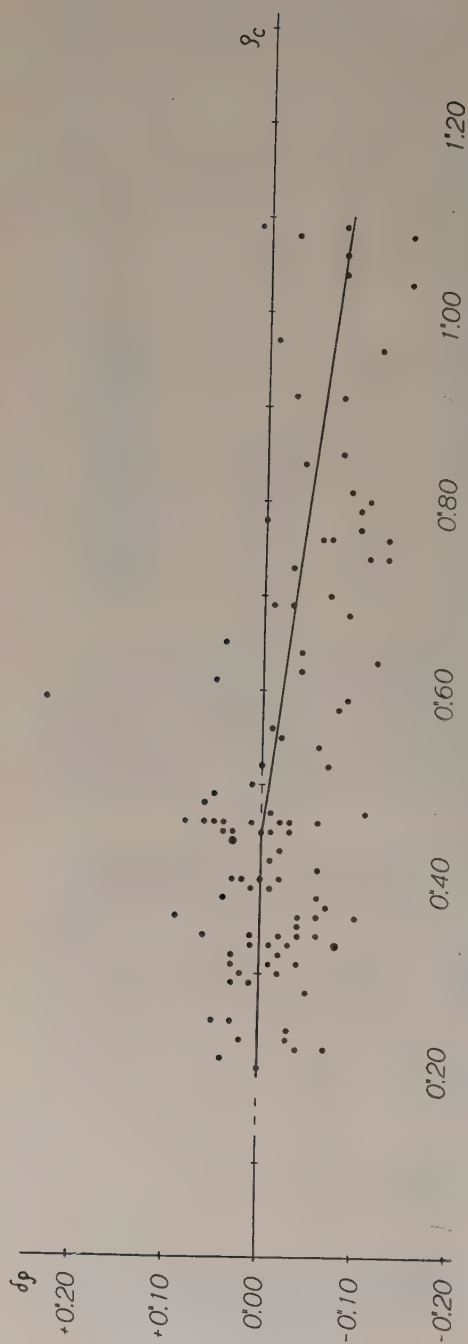


Fig. 5. Comstock. Dependence of the errors in distance on the computed distance.

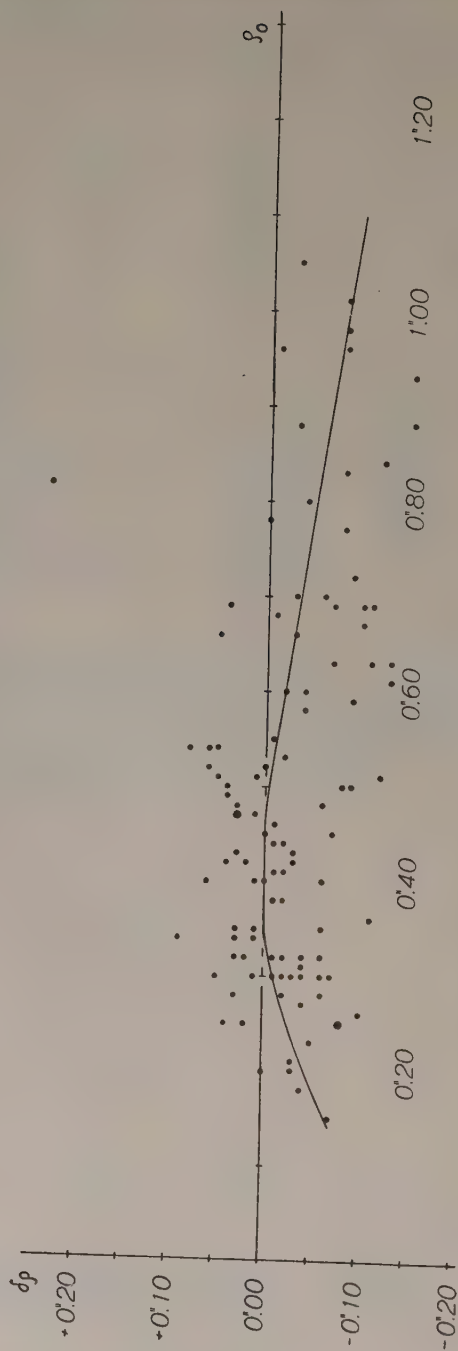


Fig. 6. Comstock. Dependence of the errors in distance on the observed distance.

obtained from direct computation. This has been purposely done in order to avoid overcorrection. — In any case, the mean values of the errors are numerically small and rather irregular. The dispersions are also irregularly distributed; the large dispersion for 45° is, however, noticeable.

Robert Grant Aitken.

Robert Grant Aitken (* 1864) made his double star observations from 1895 to 1935, when he retired from active service. The instruments used were the 12-inch (theoretical resolution limit $0''.45$) and the 36-inch (theor. resolution limit $0''.15$) refractors of the Lick Observatory; the former instrument was used only during his first years of observing. — Nearly all his observations have been published in the *Publications* and the *Bulletins of the Lick Observatory*.²⁸

The data for the stars used in the investigation are summarized in Table 11.

Table 11. *Aitken. Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\bar{\delta}_\theta$	σ_θ	$\bar{\delta}_\rho$ (in $0''.01$)	σ_ρ (in $0''.01$)
	m		$^\circ$	$^\circ$		
520	0.2	17	$+0.5 \pm 0.8$	3.4	-0.2 ± 1.0	4.3
6993	1.5	20	$+2.8 \pm 0.6$	2.8	-0.5 ± 0.6	2.6
7284	0.3	8	-0.6 ± 0.4	1.2	-0.8 ± 3.1	8.7
8189	0.4	4	$+1.7 \pm 0.5$	1.0	$+6.3 \pm 3.3$	6.6
8987	0.0	22	$+3.6 \pm 0.8$	3.7	-1.2 ± 0.7	3.1
9247	1.7	20	-0.9 ± 0.7	3.0	$+0.5 \pm 0.4$	1.9
9378	0.5	5	-0.6 ± 0.7	1.6	$+0.6 \pm 1.2$	2.6
9617	0.5	7	-0.7 ± 0.3	0.7	-4.0 ± 3.0	8.0
10157	3.5	12	-0.7 ± 1.2	4.0	-1.6 ± 2.4	8.4
—34°6803	2.0	12	-0.4 ± 0.2	0.7	-7.7 ± 4.1	14.3
11871	3.5	25, 24	-1.4 ± 0.4	2.0	-2.0 ± 2.0	9.6
11950	1.0	20	-1.9 ± 1.0	4.6	$+1.6 \pm 1.0$	4.7
12973	1.0	10	$+3.0 \pm 1.8$	5.7	-0.2 ± 0.4	1.3
14499	0.5	7	-2.6 ± 2.0	5.2	$+1.0 \pm 1.1$	2.9
15176	0.1	7, 8	$+0.6 \pm 1.0$	2.7	-1.4 ± 2.0	5.8
16665	1.0	21	$+0.5 \pm 0.6$	2.6	-0.4 ± 0.9	4.3

In the table there is an excess of negative values of $\bar{\delta}_\rho$: 11 negative values against 5 positive. However, the numerical values are generally small.

In the continuation the observations made with the 12-inch and those made with the 36-inch refractor have been separated. Since the number

²⁸ Lick Publ 12 (1914); Lick Bull 11, p. 58 (1923); Lick Bull 12, p. 173 (1927); Lick Bull 14, p. 62 (1929); Lick Bull 16, p. 96 (1933); Lick Bull 17, p. 91 (1935); Lick Bull 18, pp. 53, 109 (1937). Of the 25 lists of new double stars, the first two were published in the *AN* and the others in the *Lick Bull*.

of observations with the 12-inch is rather limited, these observations generally have not been further investigated. Therefore the following results apply only to the observations with the 36-inch, unless otherwise is stated.

Position angles. When the results from different epochs are compared, some systematic differences are found, but these are probably not real. — The results obtained with the 12-inch have been compared with those obtained with the 36-inch during the same period (1896—1904), and the difference between the means of the errors (12-inch *minus* 36-inch) has been found to be:

$$-1^{\circ}.5 \pm 0^{\circ}.6$$

Nearly all the position angles turn out to be measured too small with the 12-inch.

No regular dependence on the distance has been found; the values of the dispersion decrease in a regular way when the distance increases.

The results for the eight position angle sectors are given in Table 12.

Table 12. *Aitken, 36-inch. Position angles. Errors for different position angle sectors.*

Sector	N	$\bar{\delta}_\theta$	σ_θ
0°	13	$+1.1 \pm 0.9$	3.1
45	22	-0.3 ± 0.7	3.3
90	24	-1.3 ± 0.7	3.4
135	18	-0.2 ± 0.7	2.8
180	36	$+1.6 \pm 0.7$	4.1
225	31	-0.1 ± 0.6	3.3
270	36	$+1.3 \pm 0.5$	2.8
315	10	$+1.1 \pm 1.2$	3.7

If there is any dependence on the position angles, this is at least not very conspicuous.

Distances. As in the case of position angles, a comparison has been made between the results obtained with the 12-inch and with the 36-inch refractor. Since such a comparison has already been carried out by F. SCHLESINGER,²⁹ the present comparison has been made only between the mean values of the errors during the period 1896—1904, without making any division with respect to the distances. The distances are on an average a little below 1". The mean values of the errors during the time mentioned are (in 0".01):

²⁹ Science, NS 44, p. 571 (1916); see also R. G. AITKEN, *The Binary Stars*, p. 63—64 (1935).

with the 12-inch: $+3.6 \pm 1.6$

» » 36-inch: -1.3 ± 0.7

giving the following difference (12-inch *minus* 36-inch):

$$+4.9 \pm 1.7$$

This is in good agreement with Schlesinger's results. According to the above figures, the difference is mainly due to the fact that Aitken measured these small distances too large with the 12-inch refractor.

The dependence on time, distance and position angle has been studied only for the 36-inch refractor.

No appreciable variations with the time of observation have been found.

When the errors in distance are plotted against the computed distances, it is obvious that the smallest distances ($< \text{about } 0''.20$) have generally been measured too large. In order to test this result further, the values of the errors for computed distances smaller than $0''.30$ have been added for the following double stars: ADS 3210, 8337, 9094, 9744, 9769, 11111, 12145, 15281 and 16800. The number of annual means used has thus been increased to 307. Fig. 7 shows that the above conclusion is corroborated: for the smallest distances there is a distinct excess of positive values, while there is an excess, although not so obvious, of negative values when the distance becomes larger. If corrections according to the mean curve in Fig. 7 are applied, the sum of the absolute values of the residuals is reduced by 12 %. — Fig. 8 shows that the dependence is far less pronounced when the observed distance is taken as abscissa. The corrections which may be derived from the mean curve in Fig. 8 cannot be supposed to be very reliable. Thus, there undoubtedly exist errors depending on the distance, but these errors act in such a way that it does not unfortunately seem possible to correct for them.

Previous to the investigation of the dependence on the position angle, the values of the errors have been corrected with respect to the distance by aid of the mean curve in Fig. 7. From the stars given in Table 11 the following results are thus found for the different sectors. (See Table 13.)

Table 13. *Aitken, 36-inch. Distances. Errors for different position angle sectors. (Values corrected for distance.)*

Sector	N	$(\delta\rho)_2$ (in $0''.01$)	$(\sigma\rho)_2$ (in $0''.01$)
0°	13	$+1.8 \pm 1.6$	5.9
45	22	-3.8 ± 1.4	6.7
90	24	-0.6 ± 1.0	5.1
135	18	-1.9 ± 1.1	4.7
180	36	-0.1 ± 0.6	3.5
225	31	-1.7 ± 0.7	3.7
270	36	-1.7 ± 0.9	5.6
315	10	$+0.6 \pm 1.3$	4.1

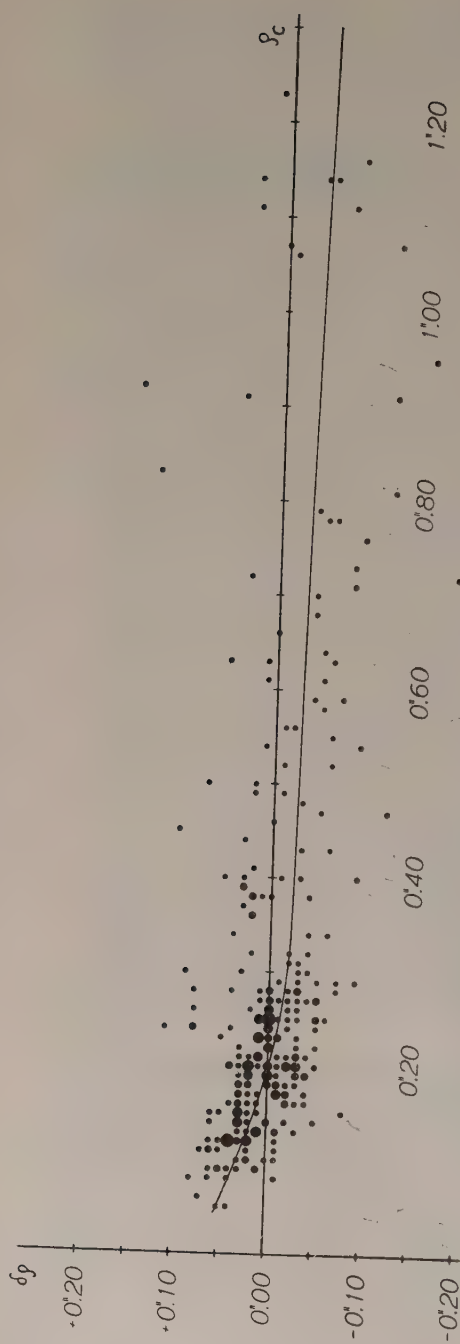


Fig. 7. Aitken, 36-inch. Dependence of the errors in distance on the computed distance.

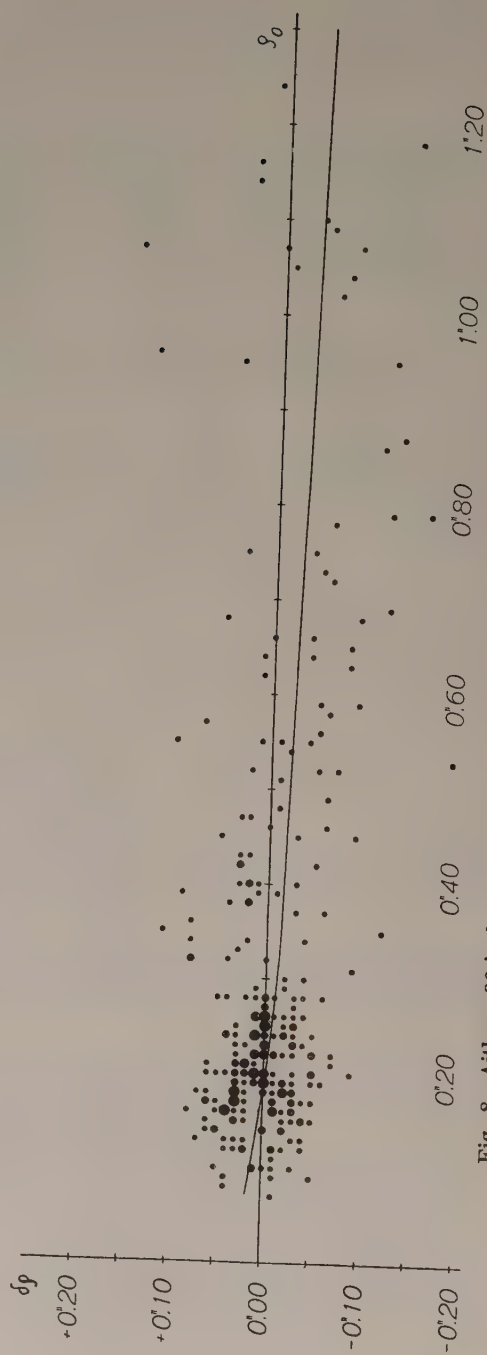


Fig. 8. Aitken, 36-inch. Dependence of the errors in distance on the observed distance.

The symbols in this table are the same as in Table 10.

There is an excess of negative values of $(\overline{\delta\rho})_2$, which may be explained in the same way as in the case of Comstock. — A slight dependence on the position angles seems to be present, although the effects appear to be rather irregular. The numerically large negative value of $(\overline{\delta\rho})_2$ for 45° is noteworthy.

William Bryant.

William Bryant (* 1865, † 1923) began his double star observations at the Greenwich Observatory in 1897 and continued them until shortly before his death. The instrument that he used was the 28-inch refractor (theoretical resolution limit $0''.19$). The measurements obtained with this instrument are generally not of so high a quality as might be expected from its large size; it is also known that the mounting of this instrument is uncomfortable.

Bryant's measurements, as well as those of the other Greenwich observers, have been published in the annual volumes of the *Greenwich Observations*, and in the *Greenwich Catalogue of Double Stars*.³⁰ In the latter volume the results are not given separately for each observer. It should be noticed that several measurements made by Bryant are to be found in the volumes of the *Greenwich Observations* published after his death.³¹

Table 14 gives the data for the stars used in this investigation.

From the table it is immediately seen that Bryant generally measured the distances too small (15 negative and only 3 positive values of $\overline{\delta\rho}$). — There also seems to be a slight excess of negative values of $\overline{\delta\theta}$.

In this table the position angle values of ADS 684 have been excluded because of the abnormal sizes of the errors. Likewise, some single values (both in position angle and in distance) have been rejected.

Position angles. No regular dependence of the errors in position angle on time or distance has been found.

Table 15 gives the results for the different position angle sectors.

The most characteristic feature in this table seems to be the large values of the dispersion in the directions 0° , 180° and 225° . From this it may be inferred that Bryant's measurements of position angles are less reliable in the vertical direction than in other directions. It must, however, be admitted that the material is rather small for an investigation of this kind.

³⁰ Catalogue of Double Stars from observations made at the Royal Observatory, Greenwich, with the 28-inch Refractor during the years 1893—1919, p. 49—229. London (1921).

³¹ Greenwich Observations 1920, 25, 27, 30, 34 and 35.

Table 14. *Bryant. Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\bar{\delta}_\theta$	σ_θ	$\bar{\delta}_\rho$ (in $0''.01$)	σ_ρ (in $0''.01$)
	III					
684	0.5	0, 3	$+ 1.5 \pm 0.5$	1.2	$- 0.7 \pm 7.1$	12.2
1709	0.9	6	$+ 0.4 \pm 0.7$	1.8	$- 10.7 \pm 3.1$	7.5
5871	0.0	6	$- 1.5 \pm 3.2$	7.9	$- 12.1 \pm 4.2$	10.3
6483	0.2	6	$- 0.4 \pm 0.7$	1.6	$- 6.8 \pm 2.8$	6.9
7284	0.3	5	$- 1.2 \pm 1.2$	2.7	$- 1.8 \pm 1.1$	2.4
7390	0.8	5	$- 2.8 \pm 3.1$	5.3	$- 15.0 \pm 4.0$	9.0
8189	0.4	3	$- 0.3 \pm 2.7$	8.0	$- 7.0 \pm 1.4$	2.5
8987	0.0	9	$+ 10.0 \pm 2.8$	7.5	$+ 1.0 \pm 1.8$	5.3
9229	0.1	7, 15	$- 9.3 \pm 2.7$	4.7	$+ 3.7 \pm 1.7$	6.7
9247	1.7	3	$- 1.4 \pm 1.0$	3.1	$+ 2.7 \pm 1.6$	2.8
9378	0.5	9, 8	$- 1.6 \pm 1.0$	3.5	$- 7.3 \pm 1.3$	3.6
10157	3.5	11	$- 0.2 \pm 0.8$	1.9	$- 11.2 \pm 1.7$	5.5
10871	0.2	6	$+ 5.3 \pm 6.1$	15.0	$- 2.7 \pm 1.1$	2.8
11111	1.5	6	$- 5.1 \pm 2.5$	7.9	$- 2.3 \pm 4.4$	10.7
12973	1.0	10, 9	$- 4.5 \pm 1.0$	3.2	$- 1.1 \pm 2.3$	6.8
14499	0.5	10, 11	$+ 1.9 \pm 0.9$	2.4	$- 1.1 \pm 2.2$	7.2
16173	0.0	8	$+ 7.3 \pm 3.6$	10.1	$- 5.1 \pm 2.2$	6.1
16665	1.0	8			$- 4.3 \pm 3.2$	9.1

Table 15. *Bryant. Position angles. Errors for different position angle sectors.*

Sector	N	$\bar{\delta}_\theta$	σ_θ
0°	8	$+ 3.9 \pm 3.8$	10.8
45	17	$+ 1.1 \pm 1.1$	4.4
90	28	$+ 2.5 \pm 0.9$	5.0
135	21	$- 3.7 \pm 1.0$	4.5
180	13	$- 0.4 \pm 2.1$	7.6
225	13	$+ 1.0 \pm 2.7$	9.9
270	13	$- 4.1 \pm 0.8$	2.7
315	5	$+ 0.9 \pm 0.6$	1.4

Distances. The errors in distance have been plotted against the time. There is hardly any evidence of a dependence. — Since the errors turn out to depend very much on the distance (see below), they have been corrected for distance effect by the application of the corrections corresponding to Fig. 9. When the errors in the distances corrected in this way are plotted against the time, it is found that the deviation from the zero line are of an altogether accidental character. It is, however, conspicuous that the dispersion is considerably larger during the first few years (before 1904) than in the later years.

The dependence on the computed distances is shown in Fig. 9. In spite of the considerable accidental errors it is easily perceived that the smallest distances (to about $0''.25$) are measured too large, while the larger distances are measured too small. The mean curve has been approximated mainly by two straight lines. If the corresponding corrections are applied, the sum

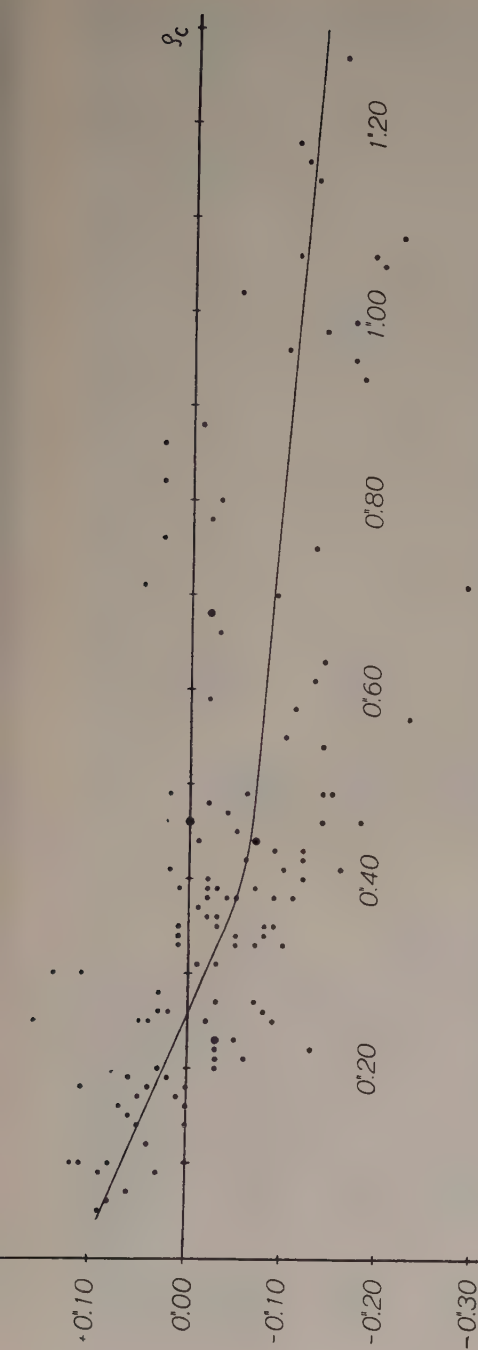


Fig. 9. Bryant. Dependence of the errors in distance on the computed distance.

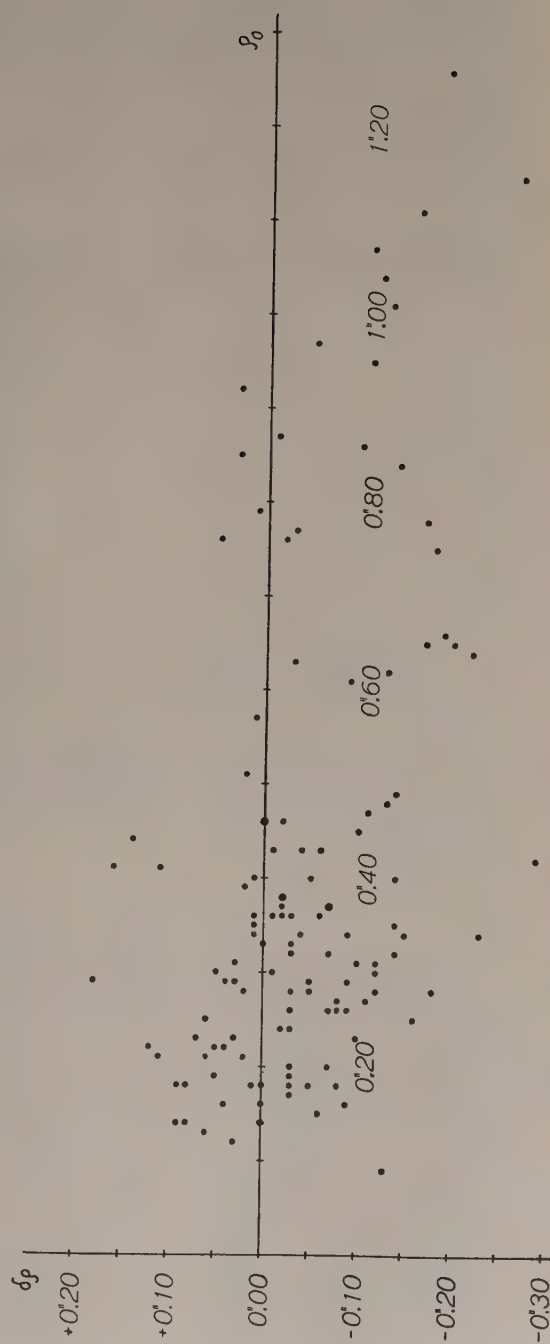


Fig. 10. Bryant. Dependence of the errors in distance on the observed distance.

of the absolute values of the residuals is reduced by 29 %. — On the other hand, the effect is largely washed out if the observed distance is taken as abscissa. (Fig. 10.) It has therefore not been attempted to draw a mean curve in this case.

The results for the different position angle sectors (values corrected with respect to the distance according to Fig. 9) are given in Table 16.

Table 16. *Bryant. Distances. Errors for different position angle sectors. (Values corrected for distance.)*

Sector	N	$(\bar{\delta\rho})_2$ (in 0".01)	$(\sigma\rho)_2$ (in 0".01)
0°	9	— 0.3 ± 2.6	7.7
45	18	+ 0.5 ± 1.2	5.3
90	33	— 1.0 ± 1.0	5.5
135	20	— 2.4 ± 1.5	6.9
180	12	— 2.2 ± 1.0	3.4
225	13	+ 2.5 ± 1.8	6.4
270	16	+ 3.2 ± 1.5	6.2
315	7	— 0.6 ± 3.8	10.0

The symbols are the same as in Table 10.

Possibly the distances are measured too large in the approximate directions 45° and 225° (and 270°), and too small in the directions perpendicular to these, but the mean errors are so large that no certain conclusions can be drawn. The dispersions in the table vary rather irregularly; there are large variations, but it is difficult to decide whether they are real or not.

George van Biesbroeck.

The first series of measurements by George van Biesbroeck (* 1880) dates from 1903—04 and was carried out at Uccle with the 15-inch refractor (theoretical resolution limit 0".36). After a comparatively short series of measurements with the Heidelberg 12-inch refractor he went back to Uccle, where he continued observing with the 15-inch until 1915. Then he went over to the Yerkes Observatory, where his double star observations have since then been carried out with the 40-inch refractor (theor. resolution limit 0".14), at present the largest instrument that is used for double star work.

The observations made by van Biesbroeck at Uccle have been published in the *Annales de l'Observatoire Royal de Belgique*,³² those made at Heidel-

³² *Annales de l'Observatoire Royal de Belgique*, NS 9, p. 1 (1904); 13, p. 339 (1914); 14, p. 239 (1920).

berg, in the *Astronomische Nachrichten*;³³ and those made at the Yerkes Observatory, in the *Publications of the Yerkes Observatory*.³⁴ — It ought to be mentioned that the dispersion for different separations in the last series of measurements at the Yerkes Observatory has been given in a note by F. SCHLESINGER.³⁵ This note also contains several interesting general remarks on the accuracy of double star measurements.

In the observations of the position angles of close pairs (distance below 2") made at Uccle since May, 1911, the method of Salet and Bosler has always been applied. The same method has also nearly always been practised at the Yerkes Observatory since February, 1923.

In his observations during his last years at Uccle, van Biesbroeck gradually adopted the way of observing the position angles having the micrometer threads perpendicular to the line joining the two components, instead of the usual way (threads parallel to the line joining the stars). The method of placing the threads normal to the line between the stars has been used in all the work made with the 40-inch refractor, except for large angular distances ($>20''$).

It will further be of interest to note that van Biesbroeck's left eye, which is always used in his observing, is slightly astigmatic. Since 1918 this defect has been corrected by means of a lens with the characteristics, Sph — 3 D Cyl. — 0.50 D. Ax. 150° .³⁶

A summary of the results for each star is given in Table 17.

Table 17. *Van Biesbroeck. Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\bar{\delta}_0$	σ_0	$\bar{\delta}_\rho$ (in $0''.01$)	σ_ρ (in $0''.01$)
	m		$^\circ$	$^\circ$		
520	0.2	3	$+5.6 \pm 6.6$	11.5	$+3.3 \pm 3.2$	5.6
5871	0.0	15	$+0.3 \pm 0.4$	1.7	-8.6 ± 1.7	6.5
6993	1.5	5	$+1.9 \pm 2.0$	4.6	$+0.8 \pm 0.8$	1.7
7284	0.3	13	-0.9 ± 1.1	4.0	$+0.5 \pm 1.1$	4.1
8189	0.4	8	$+1.4 \pm 0.8$	2.3	-1.1 ± 0.6	1.8
8987	0.0	10, 9	-0.9 ± 2.1	6.6	$+1.9 \pm 1.1$	3.2
9247	1.7	6	$+4.7 \pm 2.4$	5.8	$+2.3 \pm 1.4$	3.5
9378	0.5	7	$+0.6 \pm 0.8$	2.2	$+3.6 \pm 0.9$	2.3
9617	0.5	13	0.0 ± 0.3	1.2	-0.9 ± 2.2	8.0
10157	3.5	9	$+0.8 \pm 0.5$	1.5	-8.2 ± 2.5	7.5
11871	3.5	20, 19	$+1.5 \pm 0.7$	3.3	-1.9 ± 2.2	9.7
12973	1.0	11	-1.0 ± 1.9	6.2	$+2.5 \pm 1.5$	4.9
14499	0.5	14, 12	$+6.8 \pm 2.1$	8.0	-0.2 ± 1.4	4.8
15176	0.1	6	$+0.9 \pm 2.3$	5.7	-0.2 ± 0.8	1.9
16665	1.0	10	$+0.8 \pm 2.3$	7.2	-3.1 ± 1.6	5.1

³³ AN 172, col. 33 (1906).

³⁴ Yerkes Publ 5:1 (1927); 8:2 (1936).

³⁵ PASP 49, p. 39 (1937).

³⁶ Cf. Yerkes Publ 5:1, p. 4.

The excess of positive values of $\bar{\delta}_0$ is rather unexpected. Since almost half of the observations were carried out according to Salet and Bosler's method, it might be expected that the excess of positive values depends on the observations in which the prism method was not used. This is, however, found not to be the case when the two kinds of observations are treated separately. Possibly, the explanation may be that van Biesbroeck, like several other observers, has carried out the observations with the line through the stars either parallel or perpendicular to the line joining the eyes. As a consequence of this, he has had to incline his head when observing, and, as has already been pointed out,³⁷ this may have caused systematic errors. However, considering the limited material, it is also possible that the preponderance of positive values is simply due to chance.

In the continuation, the observations with the 15-inch and with the 40-inch refractor have been treated separately.

Position angles. As mentioned above, the material is not homogeneous with respect to the methods of observing the position angles. Since the material seems too scanty to permit a division according to different observation methods, the further investigation has not been carried into detail. It ought, however, to be mentioned that no certain variations with the time have been established, and that the dispersion in the errors decreases regularly when the distance increases.

Distances. The variations with the time, if any, do not seem to be of importance.

In Fig. 11, the dependence of the errors on the computed distance is given for the observations with the 15-inch refractor. The number of annual means is 70. The mean curve has a very characteristic run, similar to that of the corresponding curves for Aitken's and especially Bryant's observations. — If corrections according to the mean curve are applied, it is found that the sum of the absolute values of the residuals is reduced by 30 %.

Similarly, Fig. 12 shows the same relation for the 40-inch (from 75 annuals means). When this figure is compared with the previous one, it is interesting to see that the systematic errors are much smaller with the larger instrument, while the accidental errors have by far not been reduced to the same extent. In spite of the fact that the systematic errors have been largely removed, the mean curve has undoubtedly a run similar to that found in the former case. — By applying corrections in the same way as previously, a reduction of 11 % is obtained.

The dependence on the position angle has not been investigated, since the material seems to be too small.

³⁷ See p. 9.

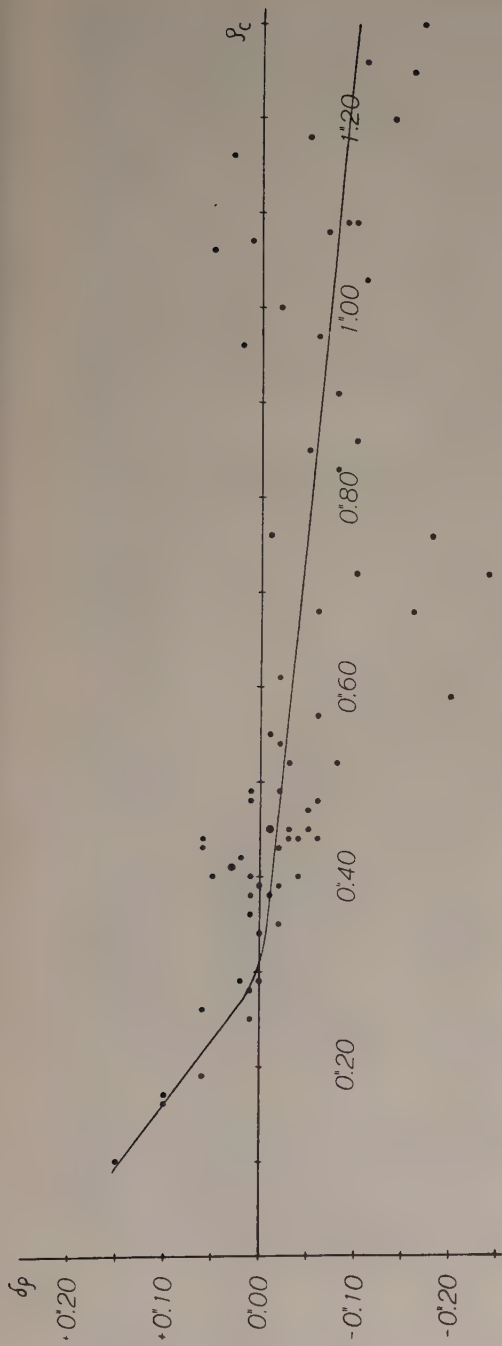


Fig. 11. Van Biesbroeck, 15-inch. Dependence of the errors in distance on the computed distance.

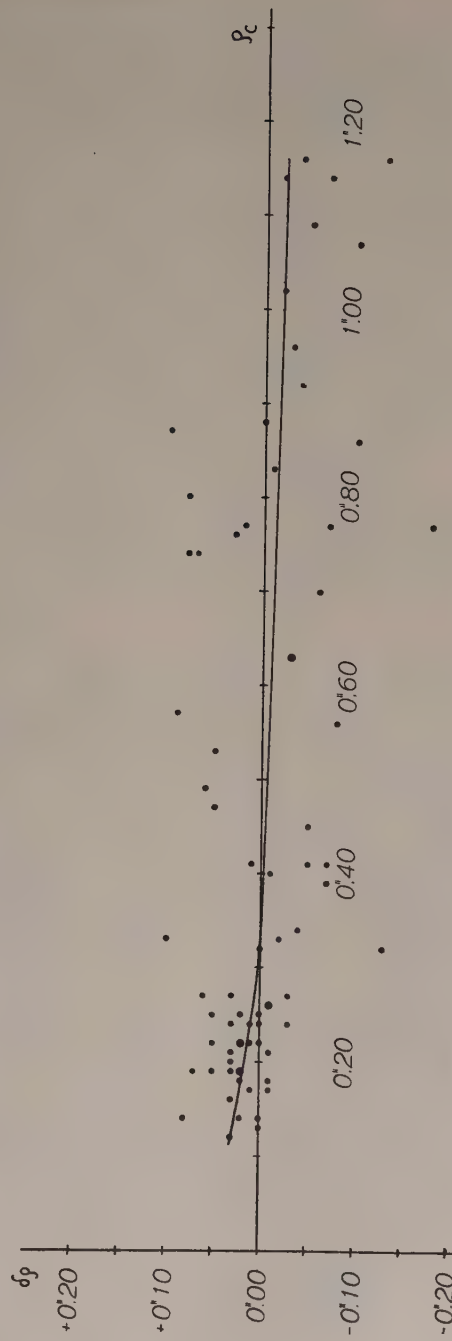


Fig. 12. Van Biesbroeck, 40-inch. Dependence of the errors in distance on the computed distance.

Willem H. van den Bos.

The double star observations made by W. H. van den Bos (* 1896) began in the year 1920 at the Leiden Observatory. His observations there were made with the 10 $\frac{1}{2}$ -inch refractor (theoretical resolution limit 0".52) and continued until 1925, when he went to the Union Observatory, Johannesburg, S.A. Here his measurements have generally been made with the 26 $\frac{1}{2}$ -inch refractor (theor. resolution limit 0".21); some measurements have, however, been carried out with the 9-inch refractor.

The Leiden observations have been published in the *Annalen van de Sterrewacht te Leiden*.³⁸ The Johannesburg visual observations are to be found in the above-mentioned Annals³⁹ and in the *Union Observatory Circulars*.⁴⁰ His lists of new double stars have been published in the *Bulletins of the Astronomical Institutes of the Netherlands*.

The position angle measurements made at Leiden and generally also those made at Johannesburg have, whenever possible, been carried out with the help of a Salet and Bosler prism.

In the following table (Table 18) only observations with the 10 $\frac{1}{2}$ -inch and the 26 $\frac{1}{2}$ -inch have been included.

Table 18. *Van den Bos. Stars used in the comparison; results for each star.*

ADS or CPD	Δm	N	$\overline{\delta_\theta}$	σ_θ	$\overline{\delta_\rho}$ (in 0".01)	σ_ρ (in 0".01)
	m					
520	0.2	10	-2.2 ± 0.7	2.2	$+ 0.4 \pm 0.7$	2.2
7284	0.3	3	-6.0 ± 2.4	4.2	$+ 11.7 \pm 1.8$	3.1
48°4965	0.0	15	0.0 ± 0.4	1.6	$- 0.5 \pm 0.4$	1.7
8987	0.0	3	-1.7 ± 2.6	4.5	$- 3.0 \pm 1.4$	2.5
9617	0.5	4	-0.7 ± 0.8	1.7	$- 1.3 \pm 1.5$	3.0
11871	3.5	5	-0.2 ± 0.1	0.3	$- 4.0 \pm 2.5$	5.6
11950	1.0	11	$+ 0.4 \pm 0.4$	1.3	$+ 0.5 \pm 0.9$	2.9
14499	0.5	3	$+ 2.6 \pm 2.0$	3.4	$+ 4.7 \pm 3.1$	5.3
15176	0.1	3	-4.7 ± 2.6	4.4	$+ 1.0 \pm 1.9$	3.3
16665	1.0	6	$+ 0.9 \pm 0.6$	1.4	$+ 1.2 \pm 2.5$	6.1

From the table it is seen that the mean values are as a rule numerically small and well distributed with respect to the signs. It should be noticed that the large value of $\overline{\delta_\rho}$ for ADS 7284 has been obtained from only 3 annual means; these observations have, moreover, been made with the 10 $\frac{1}{2}$ -inch.

³⁸ Leiden Ann **14:3** (1925).

³⁹ Leiden Ann **14:4** (1928).

⁴⁰ UOC **78**, p. 13 (1928); **80**, p. 59 (1929); **83**, p. 135 (1930); **85**, p. 183 (1931); **87**, p. 273 (1932); **88**, p. 324 (1932); **90**, p. 401 (1933); **92**, p. 61 (1934); **94**, p. 149 (1935); **96**, p. 234 (1936); **98**, p. 362 (1937); and **100**, p. 481 (1938).

In the continuation the measurements obtained with the 10 $\frac{1}{2}$ -inch (14 annual means) and with the 26 $\frac{1}{2}$ -inch (49 annual means) have been treated separately. The limited material permits only a superficial study of the errors. A careful study of the errors in the Leiden measurements has been carried out by van den Bos himself in his dissertation;⁴¹ see also the introduction to the Leiden measurements.⁴²

Position angles. The small differences found between the results obtained with different instruments or at different times of observation do not seem to be of any importance.

The dependence on the distance has been investigated for the 26 $\frac{1}{2}$ -inch only; as could be expected, there does not seem to be any dependence of this kind. The decrease of the dispersion with increasing distance is of a regular character.

The dependence on the position angle has not been investigated.

Distances. There seems to be a slight difference between the errors in the earlier and the later measurements with the 26 $\frac{1}{2}$ -inch in the sense that the earlier measurements give somewhat larger distances. The difference is, however, small and of doubtful significance.

Fig. 13 shows how the errors in the measurements with the 26 $\frac{1}{2}$ -inch refractor depend on the computed distances. The mean curve in this figure has been drawn only from 0".10 to about 0".65; for values larger than 0".40 it coincides with the zero axis. The excess of positive values for the smallest distances (< about 0".15) is evident, but for other values the curve could almost as well have been drawn along the zero axis. On the whole, the measurements made by this observer appear to be remarkably free from systematic errors, and at the same time the accidental errors are small. This statement applies to the position angle measurements as well as to the distance measurements.

The corresponding values obtained with the 10 $\frac{1}{2}$ -inch refractor are too few for a close examination; it may, however, be mentioned that the errors in the smallest distances are positive and larger than those obtained with the 26 $\frac{1}{2}$ -inch for the same distances.

No investigation of the dependence of the errors in distance on the position angles has been made.

⁴¹ »Micrometermetingen van dubbelsterren». Diss. Leiden (1925).

⁴² Leiden Ann 14:3, p. 5—16 (1925).

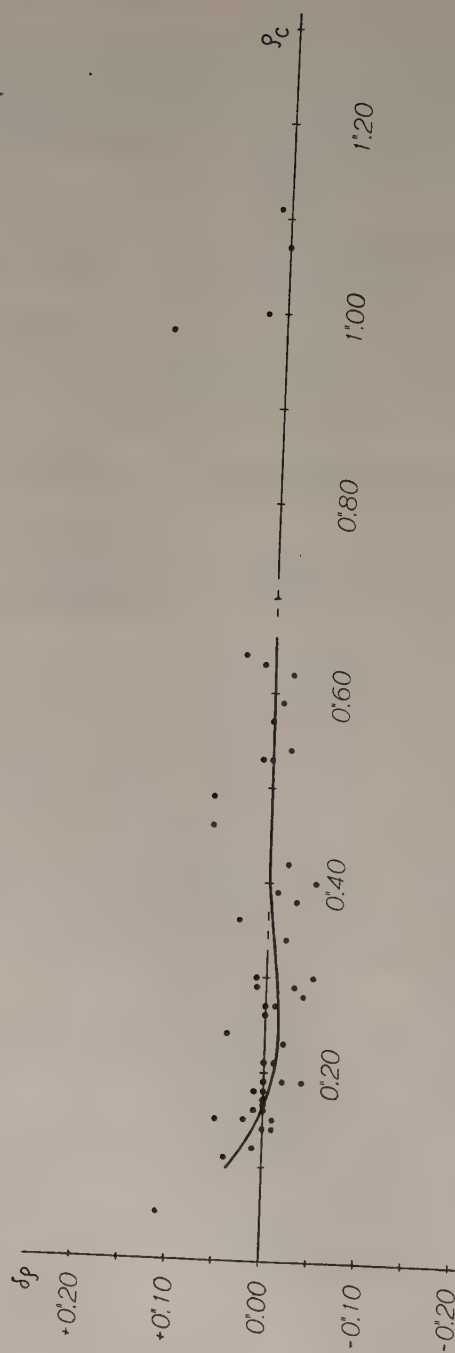


Fig. 13. Van den Bos, 26 $\frac{1}{2}$ -inch. Dependence of the errors in distance on the computed distance.

Summary and discussion of the results.

Several of the results obtained for each observer in the foregoing account are either rather uncertain or of a secondary interest. It therefore seems appropriate to give a summary of the more important results. At the same time there will be mentioned some conclusions which can be drawn from the material and which have not been treated before. In the summary the results will be arranged according to the different arguments that are to be considered, and some general reflections will also be added.

The variability of the errors with time has been investigated for all the observers, and there seems to be reason to assume that the material has generally been sufficiently large for the detection of considerable changes of a secular or longperiodic character. In spite of this, such changes have been established to a certainty only in the case of Otto Struve, for whom they have been found to be present both in position angle measurements and in distance measurements. Among the other observers, Comstock and van den Bos show doubtful changes in the measurements of distance, but all the other results are negative.

When the errors in the position angle measurements for different distances are compared, there hardly seems to be any large differences as far as the mean values are concerned. On the other hand, there is nearly always a decrease in the dispersion (expressed in degrees) when the distance increases. The only exception is Otto Struve, whose results are rather irregular and hardly show any clear change in the dispersion values, except for the very small distances. — Of course, it is quite natural that the dispersion ought to decrease when the distance increases. If it is assumed that the dispersion is inversely proportional to a power of the computed distance, the value of the exponent may be estimated by means of a graphical representation, using logarithmic scales. The values of the exponent seem to be rather different for different observers, but if the distance is not so small that the images are in contact, then the assumption

$$\sigma_{\theta} \sim \frac{1}{\rho_c^{0.8}}$$

often seems to give a fair approximation. (σ_{θ} =the dispersion, expressed in degrees; ρ_c =the computed distance.) If the components are in contact, the decrease of the dispersion with increasing distance is more rapid. — If the dispersion expressed in arc is σ''_{θ} , the above expression corresponds to:

$$\sigma''_{\theta} \sim \rho_c^{0.2}$$

The mean curves for the error in distance as a function of the computed distance are given in Fig. 14 for the following observers: Otto Struve (OS),

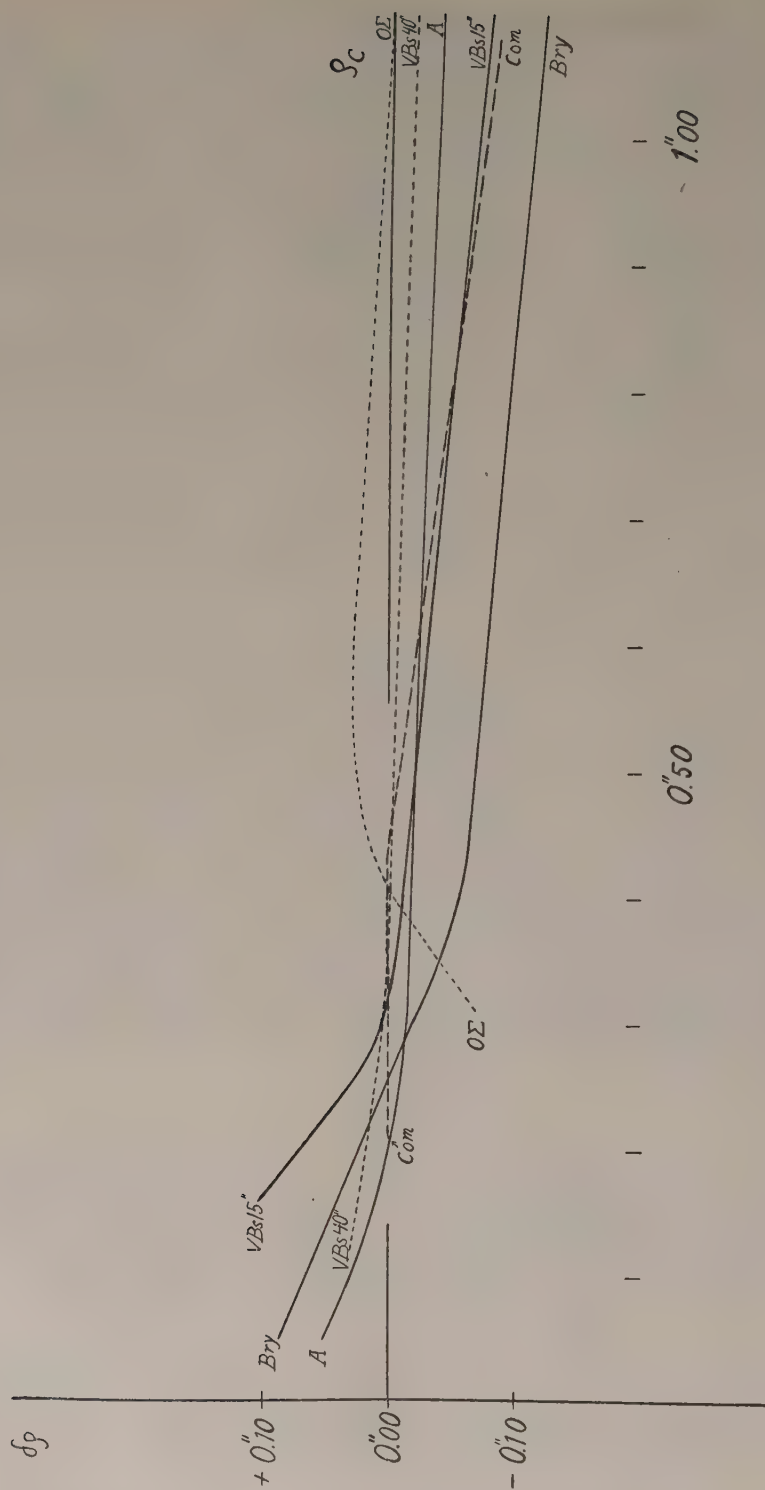


Fig. 14. Dependence of the errors in distance on the computed distance (curves for different observers).

Comstock (Com), Aitken (A), Bryant (Bry), and van Biesbroeck (VBs). The run of the curves is rather similar, except Comstock's and especially Otto Struve's.

It is curious to note that just Otto Struve, who has laid down so much work on determining systematic errors, deviates in so many respects from most other observers.

To judge from Fig. 14, there seems to exist a common tendency (although exceptions are to be found) to measure the distances too large if they are very small (or, approximately, below the resolution power). On the other hand, the larger distances seem to be measured too small. It must not, however, be overlooked that these »larger distances» are still to be considered as small, as compared with those which generally appear in double star measurements. The above result thus applies only to distances up to a little more than $1''$.

The above-mentioned statement is in good agreement with the fact that small distances are generally measured too large with small instruments, as has often been noticed, *e.g.* by P. BAIZE.⁴³ — It is also in accordance with the results obtained above for the observers (Aitken, van Biesbroeck, van den Bos), for whom observations with two different instruments are available. In all the three cases it has been found that the smallest distances are measured larger with the smaller instruments than with the larger ones. There are, however, also observers for which an opposite effect is present, as in the case of Ph. Fox. This has been pointed out by F. SCHLESINGER in an already mentioned paper.⁴⁴

Thus, even if there are exceptions to the above rule, there seems to be no doubt that it holds true in most cases, and this makes it desirable to find an explanation of it. Such an explanation has already been given by W. R. DAWES.⁴⁵ He points out that when the image of a star is bisected by a micrometer thread, the disk apparently swells out on each side of the thread. This will not cause any errors in the distance measurements as long as the separation is sufficiently large; but if the stars are not well separated, the effect will obviously be greater on the outer side of the images, and this will induce the observer to set the threads too far apart. — If the separation of the stars is below the resolution limit, the combined image will appear approximately elliptical. If in such cases the observer tries to measure or estimate the distance, this will usually come out too large, since the apparent image is more elongated than the isophotes. This has been pointed out by G. ORIANO DE VAUCOULEURS.⁴⁶

⁴³ BSAF **49**, pp. 473, 587 (1935); JO **15**, p. 96 (1932).

⁴⁴ Science, NS **44**, p. 571 (1916).

⁴⁵ Mem RAS **35**, p. 137 (1867).

⁴⁶ CR **214**, p. 762 (1942) = Publications de l'Observatoire du Houga (Gers) **4** (1942).

In this connection it ought to be remarked that the last-mentioned author also gives an account of a method for determining the distance between the components when the separation is smaller than the resolution limit. This is accomplished by means of a study of the form of the combined image of the double star. — A similar idea of measuring close double stars has earlier been used by W. H. PICKERING.⁴⁷ Pickering made use of his Scale of Ellipticities, which he had already used for measuring the ellipticity of the disks of Jupiter's satellites.

The value of the dispersion in the distance measurements as a function of the computed distance has been estimated in the same way as in the case of position angles. The results prove rather irregular in some cases, but often (as in the cases of Aitken, Bryant and van den Bos) the relation between the dispersion in distance measurements (σ_ρ) and the computed distance can be approximately expressed as follows:

$$\sigma_\rho \sim \sqrt{\rho_c}$$

As for the dependence of the errors on the position angle, the positive results are rather few. Of course, this may partly be due to the uncertainty of the method. In any case it seems probable that the dependence on the position angle, or on the angle with the vertical, is not very pronounced for most observers. Otto Struve here again seems to be an exception, since his distance measurements have been found to be clearly dependent on the position angle. — In most cases the dependence of the dispersion on the position angle does not seem to be very pronounced; in this respect Comstock's observations make a noteworthy exception.

The supposition made by Struve that opposite directions are identical with respect to the errors is in general not confirmed. In fact, the results for opposite directions do not generally even seem to be in better agreement than for any other directions.

A comparison between the dispersions in distance and in position angle (expressed in arc) has been made for each observer by computing the ratio $\frac{\sigma_\rho}{\sigma_\theta}$ for each star and estimating its mean value. Since the ratio depends on the distance, only approximate results can be obtained in this way. It may, however, be mentioned that the ratio, as could be expected, varies from one observer to another. For Otto Struve the ratio is found to be about 1.6, for Comstock and Bryant, 1.7, for Aitken, 2.4, and for van den Bos, 2.5. Thus, as a rough mean, the following value of the ratio may be adopted:

$$\frac{\sigma_\rho}{\sigma_\theta} = 2.0$$

⁴⁷ PA 22, p. 636 (1914).

It is to be observed that this value has been obtained for distances which are generally below 1". As is well known from other investigations, the ratio decreases with increasing distance, and for the widest pairs which are measured micrometrically the distance measurements generally seem to be even better than those of position angles.

Finally, a study of the influence of the magnitude difference between the stars has been made by plotting the mean values and the dispersions of the errors against the magnitude difference. If the latter is large ($>2^m.5$), it is, as might be expected, generally found that both the mean values and the dispersions are large. On the other hand, a moderate magnitude difference seems to be of surprisingly small importance, especially in angle measurements. In the distance measurements by Otto Struve and by van den Bos an increase of the dispersion with increasing magnitude difference can be distinctly traced, but in the measurements by Comstock, Aitken and van Biesbroeck no such relation can be found.

CHAPTER III

Recent work on the computation of visual binary star orbits

General remarks.

R. G. AITKEN's well-known standard work on double stars, *The Binary Stars*, which has already been mentioned here several times, contains a rather comprehensive account of some of the most important methods of computing the orbit of a visual binary.¹ After the publication of the last edition of this work in 1935 there has been issued a surprisingly large number of papers which deal, wholly or in part, with this problem. A brief account of some of the more important of these recent methods, together with references to others, will be given in the continuation. A few papers published before 1935 but not mentioned in Aitken's work will also be treated here.

The purpose is not to give a full description of the methods. Only the points of principal interest will be treated. Papers which only contain orbit determinations will not be considered, except if the paper is regarded as specially interesting from a methodical point of view. Further, the different methods will be treated mainly with regard to their practical applicability. Papers treating problems of a more special character, such as the existence of a third body in a system, will generally not be considered.

In several cases the reviewed works are accessible only with some difficulty, since they are included in relatively rare publications. On account of this, the titles of the papers will be quoted throughout in this chapter.

The different methods of computing a visual double star orbit may be roughly divided into two groups: graphical, like Zwiers' method, and analytical, like Kowalsky's and Thiele-Innes'. It ought, however, to be remarked that many orbit determinations involve both kinds of process. — Often dynamical methods are mentioned as a third group. In these orbit

¹ R. G. AITKEN, *The Binary Stars*. New York and London. 2nd Ed. (1935). Chapter IV.

methods the character of the movement, or in other words the movement as a function of the time, is of fundamental importance. The time of observation is introduced as an argument either by the law of areas or by Kepler's equation. The reason why this practice is to be recommended is evident: the time of observation is always known with a far greater degree of accuracy than the position angle and the distance between the stars. To-day an orbit method will hardly be recognized as a good one if the dynamical properties of the motion do not play a leading part, and thus nearly all of the modern methods may be said to be of a more or less dynamical character. Therefore it hardly seems appropriate to take the dynamical methods as a third group co-ordinated with the other two.

It is not easy to classify the recent works on orbit computation in a quite logical way. Since such a classification must also be considered to be of minor importance, the division in this paper has rather been made with respect to practical points of view.

Notations.

The symbols used in this chapter are in general defined in the way recommended by the International Astronomical Union.² For convenience most of the symbols have been collected below. They are briefly defined in the following list, which should be consulted when the symbols are not explained in the text.

ρ	Angular distance in the plane of the apparent orbit.
θ	Position angle.
x, y, z	Rectangular co-ordinates, referred to the apparent orbit.
r	Angular distance in the plane of the true orbit.
v	True anomaly.
M	Mean anomaly.
E	Eccentric anomaly.
t	Time of observation or instant considered; expressed in years.
τ	» » » » » » » ; » » a conveniently chosen unit and reckoned from a convenient zero epoch.
P	Period.
n	Mean angular motion.
T	Time of periastron passage.
e	Eccentricity.
φ	Eccentricity angle ($\sin \varphi = e$).

² Cf. especially Trans IAU Vol 5, 1935, p. 332, and Vol 6, 1938, p. 354.

a	Semi major axis of true orbit, in seconds of arc.	
i	Inclination.	
Ω	Position angle of the node.	
ω	Angle from node to periastron, in the plane of the true orbit.	
A	$= a (\cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i)$	Thiele-Innes constants.
B	$= a (\cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i)$	
F	$= a (-\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i)$	
G	$= a (-\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i)$	
X	$= \frac{r}{a} \cos v = \cos E - \sin \varphi$	
Y	$= \frac{r}{a} \sin v = \cos \varphi \sin E$	
c	Double areal constant, in the plane of the apparent orbit.	
A, B, C, D, E	Coefficients of the equation of the apparent ellipse: $Ax^2 + By^2 + 2Cxy + 2Dx + 2Ey = 1$	
$\mathfrak{M}_1, \mathfrak{M}_2$	Masses of components.	
p	Parallax.	
f	The universal gravitation constant.	
k^2	Attraction constant of the binary system; $k^2 = f (\mathfrak{M}_1 + \mathfrak{M}_2) p^3$.	

The index 0 denotes quantities referred to the epoch $t=t_0$ or $\tau=0$.

Derivatives with respect to the time are indicated by points, *e.g.*

$$\ddot{x} = \frac{d^2x}{dt^2} \text{ (or } = \frac{d^2x}{d\tau^2}, \text{ respectively).}$$

It has been unavoidable to use a few symbols, such as $\alpha, \beta, \gamma, \dots$, differently in the different sections of the chapter. These symbols are explained in the text.

Recent work on Thiele-Innes' method.

This method, which is now frequently used in orbit computations, was developed already in the year 1883 by TH. N. THIELE.³ In the beginning it did not attract the attention it is worth; this was probably due to the somewhat unpractical notations and formulæ used by Thiele. Since 1926 the method has been revised and extended especially by R. T. A. INNES, W. H. VAN DEN BOS and W. S. FINSEN.⁴ The method has been described in

³ Neue Methode zur Berechnung von Doppelsternbahnen. AN **104**, col. 245 (1883).

⁴ R. T. A. INNES and W. H. VAN DEN BOS: Orbital Elements of Binary Stars. UOC **68**, p. 354 (1926).

W. H. VAN DEN BOS: Orbital Elements of Binary Stars. UOC **86**, p. 261 (1932).

W. S. FINSEN: The Orbit of Hu 298. UOC **88**, p. 319 (1932).

W. H. VAN DEN BOS: Addendum to UOC No **86**. UOC **90**, p. 431 (1933).

The Binary Stars,⁵ and thus it may be sufficient here to recall the fact that the determination of the orbital elements depends on three normal places together with either the double areal constant or the period. The above data should be obtained from a careful study of the interpolation curves that give the position angles and the distances, respectively, as functions of the time.

In the *Astronomical Journal* R. H. WILSON, JR., and W. J. LUYTEN have been engaged in an interesting discussion which especially deals with the practical use of Thiele-Innes' method.⁶ The discussion refers to the double star Σ 2294=ADS 11186, whose orbit is a very narrow ellipse, a circumstance which of course makes the orbit determination more difficult. The necessary adjustment of the interpolation curves is treated in detail. The method is also compared with Zwiers' method, which Wilson considers superior in a difficult case like this.

WILSON⁷ has made the same experience when computing the orbit of π^2 Ursae Minoris=ADS 9769; this orbit is also a narrow one. He points out a difficulty in Thiele-Innes' method: the elements depend on the normal places in such a complicated way that it is hardly possible to predict the changes in the elements that will follow from a certain variation of the normal places.

Thiele-Innes' method has also been employed by F. ZAGAR⁸ in his determination of the orbits of β 456=ADS 8239 and I 365 AB=CPD -61°3841. To the determinations have also been added some general remarks on the method. Zagar *i.a.* states that other methods are to be preferred when the observations are evenly distributed along the observed arc so that several reliable normal places can be formed. On the other hand, if the number of normal places is small, then Thiele-Innes' method is to be recommended. — It might be added that the method is often suitable if there is a »gap» in the observations, as frequently occurs in close double stars when the companion is near periastron.

⁵ P. 89—95, 101—108.

⁶ R. H. WILSON, JR.: The Orbit of Σ 2294. *AJ* **43**, p. 41 (1933) = Pennsylvania Reprint **20** (1933).

W. J. LUYTEN: Note on the Orbit of Σ 2294. *AJ* **43**, p. 105 (1934).

R. H. WILSON, JR.: Revision of the Orbit of Σ 2294. *AJ* **44**, p. 71 (1935).

⁷ A new Orbit for π^2 Ursæ Minoris with Remarks on Thiele's Method. *AJ* **44**, p. 68 (1935).

⁸ Orbite delle stelle doppie visuali Burnham 456 e Innes 365. *Atti e Mem della R Accademia in Padova* **50**, 1933—34 = *Padova Pubbl* **36** (1934).

In a paper published in 1936, W. H. VAN DEN BOS⁹ gives several hints of great value to the orbit computer. The main points are given below in a somewhat abridged form.

1. In Thiele-Innes' method the three dynamical elements P , e and T are derived first, from the three normal places. When this has been done, the values of X and Y can be computed for any time t . Then the Thiele-Innes constants A , B , F and G may be computed from all the adjusted positions by the method of least squares, starting from the following two equations of condition:

$$(6) \quad \begin{aligned} XA + YF &= x \\ XB + YG &= y \end{aligned}$$

2. Thiele's fundamental equations for the three normal places may be written:

$$(7) \quad \begin{aligned} n(t_2 - t_1) - \frac{\Delta_{1,2}}{c} &= u & -\sin u \\ n(t_3 - t_2) - \frac{\Delta_{2,3}}{c} &= v & -\sin v \\ n(t_3 - t_1) - \frac{\Delta_{1,3}}{c} &= u + v & -\sin(u + v) \end{aligned}$$

Here, t_1 , t_2 and t_3 are the times of observation; $\Delta_{1,2}$ is the area of the triangle formed by the origin and the positions of the companion at the times t_1 and t_2 ; $u = E_2 - E_1$ is the difference between the eccentric anomalies at the times t_2 and t_1 ; $v = E_3 - E_2$. Other designations are explained in a similar way.

In this system c is generally assumed to be known and n unknown. It is, however, often suitable to treat c as an unknown and n as a known quantity (when the period is known from recurrence of angle). This has, for instance, been done by Finsen in his above-mentioned paper in UOC 88. Thirdly there is also the possibility that both c and n are known. The appropriate method in this case has been shown by van den Bos in another paper.¹⁰

3. In case of a narrow orbit the measurements of the position angles near minimum distance are generally subject to comparatively large errors (as expressed in degrees). It is therefore advisable to test the smoothed positions not only by the law of areas but also by means of the condition of elliptical path.

Van den Bos further gives the advice that in this case two of the three normal places ought to be chosen as near as possible to the minimum distance,

⁹ Some Remarks on Thiele's Method for computing the Orbit of a Double Star. UOC 95, p. 223 (1936).

¹⁰ The Orbit of Ho 296 = BDS 11895 = ADS 16173. UOC 90, p. 391 (1933).

one on each side of it; the third normal place should be chosen near maximum. The dynamical elements P , e and T are derived from the three normal places, but since these are not suitable for the derivation of the geometrical elements, the method given under 1 should be applied in the continuation.

4. When the elements of an orbit have been determined and an ephemeris is computed and compared with the observations, it will often be found that the elements can be improved. It is recommended to base this improvement on the angles alone, since these are in general considerably better known than the distances. With respect to the improvement of the eccentricity and the time of periastron passage the following remarks are made.

If the computed (θ, t) -curve is not sufficiently steep at the periastron, this may be corrected by increasing the value of e . A provisional value of e is adopted, and an ephemeris for the angles near periastron is computed. An improved value of e is then easily found by interpolating between the new curve and the original one.

A change in the value of T will cause a horizontal shift of the curve. This may often lead to an improvement of the representation of the values near periastron.

5. Just as changes in e and T may lead to improvements near periastron, it is frequently possible to obtain an improvement near apastron by correcting the values of Ω and a .

A change of Ω results in a vertical shift of the (θ, t) -curve. It ought to be observed that the corresponding changes in the Thiele-Innes constants can be found from the very simple formulæ:

$$(8) \quad \Delta A = -B \cdot \Delta \Omega \quad \Delta B = +A \cdot \Delta \Omega \quad \Delta F = -G \cdot \Delta \Omega \quad \Delta G = +F \cdot \Delta \Omega$$

While changes in the three above-mentioned elements are of importance especially for the position angles, a change of a will, of course, only influence the distances, by altering the scale.

6. In this paragraph van den Bos suggests a method for finding the dynamical elements if the geometrical elements have been derived first, as is generally the case in graphical methods.

It should be pointed out that several of the above remarks may, of course, also be applied to other orbit methods than that of Thiele-Innes.

A new method for deriving the Thiele-Innes formulæ has been published by S. AREND.¹¹

¹¹ Établissement, par voie raccourcie, des formules de Thiele-Innes, relatives aux orbites d'étoiles doubles, en recourant aux principes de l'affinité. Toulouse Ann **16**, p. 109 (1941).

Other analytical methods.

Of late years Laplace's idea that the computation of planetary orbits may be founded on the knowledge of the conditions in one point of the orbit has repeatedly been used for the derivation of binary orbits.

A work of this kind was published by A. FLETCHER¹² in 1931. Fletcher starts from expressions which give x , y and z as power series with respect to the time. The series are of the following type:

$$(9) \quad x = x_0 + \dot{x}_0\tau + \frac{1}{2}\ddot{x}_0\tau^2 + \frac{1}{6}\ddot{\ddot{x}}_0\tau^3 + \frac{1}{24}\ddot{\ddot{\ddot{x}}}_0\tau^4 + \dots$$

in which $\dot{x}_0$ = the value of $\frac{dx}{d\tau}$ for $\tau=0$, etc. The following relations are easily derived:

$$(10) \quad \begin{aligned} \ddot{x}_0 &= -\frac{k^2 x_0}{r_0^3} \\ \ddot{\ddot{x}}_0 &= \frac{3k^2 x_0 \dot{r}_0}{r_0^4} - \frac{k^2 \dot{x}_0}{r_0^3} \\ r_0^2 &= x_0^2 + y_0^2 + z_0^2 \\ r_0 \dot{r}_0 &= x_0 \dot{x}_0 + y_0 \dot{y}_0 + z_0 \dot{z}_0 \end{aligned}$$

If it is assumed that the values of x_0 , $\dot{x}_0$, $\ddot{x}_0$, $\ddot{\ddot{x}}_0$, y_0 and $\dot{y}_0$ are known from the observations, and that k^2 can be determined, then the above relations will successively give the values of r_0 , $\dot{r}_0$, z_0 and $z_0 \dot{z}_0$. The sign of z_0 is, however, undetermined, unless radial velocity determinations are available. Starting from the seven values: x_0 , y_0 , $\pm z_0$, $\dot{x}_0$, $\dot{y}_0$, $z_0 \dot{z}_0$ and k^2 , the orbit elements can be computed by well-known formulæ.

It might be possible to found another determination on the equations that are obtained by changing x for y and inversely. Fletcher, however, desists from this, and instead makes the determination more reliable by choosing the x -axis in nearly the same direction as the radius vector at the time $\tau=0$.

There still remains the question of determining the value of k^2 . This is achieved by means of the relation

$$(11) \quad k^2 = f(\mathcal{M}_1 + \mathcal{M}_2)p^3$$

The parallax p must thus be known. The total mass of the system can be approximated from the mass-luminosity relation. This method is certainly of great interest. However, as Fletcher himself points out, an orbit derived

¹² The Binary System 61 Cygni. MN **92**, p. 121 (1931).

in this way must naturally not be used for the determination of the mass of the system.

The method is applied by Fletcher to the orbit of 61 Cygni=ADS 14636.

L. PICART¹³ has published a paper which is of interest especially from a theoretical point of view. He assumes that x , y and their first four derivatives with respect to the time are known for a certain epoch. In the continuation he makes use of a co-ordinate system with two axes: the nodal line and the line perpendicular to it in the plane of the true orbit. By using this co-ordinate system it is easy to derive the following relation between the true distance and the co-ordinates x , y :

$$(12) \quad r^2 = \alpha x^2 + 2\beta xy + \gamma y^2$$

The coefficients α , β , γ are functions only of the elements i and Ω . By derivating this equation twice with respect to the time, the expressions of $\frac{\dot{r}}{r}$ and $\frac{\ddot{r}}{r} + \frac{\dot{r}^2}{r^2}$ can be obtained.

The differential equations of the apparent motion are:

$$(13) \quad \ddot{x} + k^2 \frac{x}{r^3} = 0 \quad \ddot{y} + k^2 \frac{y}{r^3} = 0$$

Here is

$$(14) \quad \frac{\ddot{x}}{x} = \frac{\ddot{y}}{y}$$

By derivating (13) the following relations are successively found:

$$(15) \quad \frac{3\dot{r}}{r} = \frac{\dot{x}}{x} - \frac{\ddot{x}}{\ddot{x}} = \frac{\dot{y}}{y} - \frac{\ddot{y}}{\ddot{y}}$$

$$(16) \quad \frac{3\ddot{r}}{r} - \frac{3\dot{r}^2}{r^2} = \frac{\ddot{x}}{x} - \frac{\dot{x}^2}{x^2} - \frac{\ddot{x}}{\ddot{x}} + \frac{\ddot{x}^2}{\ddot{x}^2} = \frac{\ddot{y}}{y} - \frac{\dot{y}^2}{y^2} - \frac{\ddot{y}}{\ddot{y}} + \frac{\ddot{y}^2}{\ddot{y}^2}$$

Since the above equations are to be used for the moment t_0 , the index 0 is to be implied everywhere.

From the equations it is possible to derive quantities that are proportional to α , β and γ , and from these quantities the values of Ω and i are easily obtained. The remaining elements can also be obtained from simple formulæ.

Like all similar methods, the one above is intended to be used especially if the observed arc is comparatively short. The arc must, on the other hand,

¹³ Sur le calcul des orbites des étoiles doubles visuelles. CR 198, p. 867 (1934).

have been observed with a considerable relative accuracy; the pair must thus be rather wide. Even so, practice has shown that the values of $\ddot{x}$ and $\ddot{y}$ can hardly ever be determined accurately enough. (In fact, it is often difficult even to obtain fair values of the third derivatives.) Therefore this method, like others that imply the knowledge of the fourth derivatives, is almost exclusively of a theoretical interest. — Picart points out that the equations (14)—(16) permit a control of the computation of the derivatives at an early stage.

F. ZAGAR¹⁴ has developed an analytical method which especially aims at systems of the type of 61 Cygni. In the first paragraph of his paper some general remarks on orbit methods are made. Zagar states that the purely analytical methods, which are based on the coefficients in the equation of the apparent ellipse, have the disadvantage that the epochs of observation do not enter into the computation of the five elements a , e , i , Ω and ω .¹⁵ Concerning the graphical methods, Zagar points out the arbitrariness which may occur when the curves are drawn, and on account of this he prefers methods in which least squares solutions are used.¹⁶

On account of the above considerations, Zagar develops a method in which the times of observation are taken into account from the outset, at the same time as the method of least squares is used. Zagar first assumes that the values of x , y , $\ddot{x}$ and $\ddot{y}$ are known in several points of the orbit, and derives the equations of condition in the following way.

The equation of the apparent orbit may be written:

$$(17) \quad Ax^2 + By^2 + 2Cxy + 2Dx + 2Ey = 1$$

The double areal constant is

$$(18) \quad x\dot{y} - \dot{x}y = c$$

By using the expression of $\frac{dx}{dy}$ (and $\frac{dy}{dx}$), which can be found from (17) and also from (18), the following equations are derived:

¹⁴ Sul calcolo d'orbita dei sistemi doppi del tipo 61 Cygni. *Lincei Rend* (6) **21**, p. 353 (1935) = *Padova Pubbl* **42** (1935).

¹⁵ Against this it may be objected that it is possible to take the times into account by adjusting the observed positions according to the law of areas. (Cf. p. 67.)

¹⁶ It will be of interest to compare this statement with that made by AITKEN in *The Binary Stars* (p. 175, foot-note) in his treatment of the orbits of spectroscopic binaries. Aitken does not seem to be of the same opinion as Zagar. — As will be seen later in this paper (p. 89), VAN DEN BOS does not share Zagar's opinion about the value of the method of least squares in the computation of binary orbits. Considering the nature of the observation errors, one would rather accept van den Bos' opinion. However, the method is no doubt better adapted to wide pairs than to close ones.

$$(19) \quad \begin{aligned} cAx + cCy + cD + Dx\dot{y} + Ey\dot{y} &= +\dot{y} \\ cBy + cCx - Dx\dot{x} + cE - Ey\dot{x} &= -\dot{x} \end{aligned}$$

The equations are adapted to the method of least squares by introducing the following unknowns:

$$(20) \quad X_1 = cA \quad X_2 = cB \quad X_3 = cC \quad X_4 = cD \quad X_5 = cE \quad X_6 = D \quad X_7 = E$$

This leads to the following equations of condition:

$$(21) \quad \begin{aligned} xX_1 + yX_3 + X_4 + x\dot{y}X_6 + y\dot{y}X_7 &= +\dot{y} \\ yX_2 + xX_3 + X_5 - x\dot{x}X_6 - y\dot{x}X_7 &= -\dot{x} \end{aligned}$$

The resulting two systems of normal equations can either be solved separately or be combined to one system with 7 unknowns.

When the constants A, B, C, D, E and c are known, the orbit elements can be derived.

It remains to determine the values of $\dot{x}$ and $\dot{y}$, which have been treated above as known quantities. If the observed arc is comparatively short, Zagar obtains these values by expanding x and y in power series of the time, determining the constants and derivating the series with respect to the time. If the arc is long, this process is not suitable; in this case Zagar suggests that the values of $x, y, \frac{\Delta x}{\Delta t}$ and $\frac{\Delta y}{\Delta t}$ should be plotted against the times and the necessary values be read off from the curves; $\dot{x}$ and $\dot{y}$ are supposed to be approximately equal to $\frac{\Delta x}{\Delta t}$ and $\frac{\Delta y}{\Delta t}$, respectively.

In another paper¹⁷ Zagar shows the practical application of the method on the system of 61 Cygni=ADS 14636. Since it is found that one of the constants of the apparent ellipse cannot be determined accurately enough, Zagar makes use of the mass sum and the parallax of the system in the same way as Fletcher. — The same method has also been applied by Zagar¹⁸ for the computation of a parabolic orbit of Σ 2032= σ Coronae Borealis=ADS 9979.

G. SILVA¹⁹ has developed a method which in certain respects reminds of those given by Fletcher²⁰ and Picart.²⁰ A similar idea was developed by

¹⁷ L'orbita del sistema binario 61 Cygni. Atti del R Istituto Veneto **94**:2, p. 259 (1935) = Padova Pubbl **43** (1935).

¹⁸ Orbita parabolica del sistema binario Σ 2032 (σ Coronae Borealis). Atti del R Istituto Veneto **95**:2, p. 711 (1936) = Padova Pubbl **49** (1936).

¹⁹ Su un metodo analitico per il calcolo d'orbita di una stella doppia visuale. Note I e II. Atti del R Istituto Veneto **95**:2, p. 551 (1936) = Padova Pubbl **48** (1936).

²⁰ *Loc. cit.*

K. SCHWARZSCHILD²¹ as early as 1890. — In Silva's paper the method is used for the computation of parabolic orbits of Σ 2032= σ Coronae Borealis= $=$ ADS 9979 and 61 Cygni= $=$ ADS 14636 (the latter by Miss T. SANDON; also according to Zagar's method). — The method has been applied to the system of Σ 1909=44 i Bootis= $=$ ADS 9494 by A. GENNARO.²²

Silva's method has been modified by F. ZAGAR.²³ The method will be reviewed below according to Zagar's modification.

The co-ordinates x and y are expressed in power series of the time in the same way as in Fletcher's method (equation (9)):

$$(22) \quad \begin{aligned} x &= x_0 + \dot{x}_0\tau + \frac{1}{2}\ddot{x}_0\tau^2 + \frac{1}{6}\ddot{\ddot{x}}_0\tau^3 + \frac{1}{24}\ddot{\ddot{\ddot{x}}}_0\tau^4 + \dots \\ y &= y_0 + \dot{y}_0\tau + \frac{1}{2}\ddot{y}_0\tau^2 + \frac{1}{6}\ddot{\ddot{y}}_0\tau^3 + \frac{1}{24}\ddot{\ddot{\ddot{y}}}_0\tau^4 + \dots \end{aligned}$$

By multiplying the first equation by y_0 , the second by x_0 , and subtracting, we obtain:

$$(23) \quad \begin{aligned} xy_0 - yx_0 &= (\dot{x}_0y_0 - \dot{y}_0x_0)\tau + \frac{1}{2}(\ddot{x}_0y_0 - \ddot{y}_0x_0)\tau^2 + \\ &+ \frac{1}{6}(\ddot{\ddot{x}}_0y_0 - \ddot{\ddot{y}}_0x_0)\tau^3 + \frac{1}{24}(\ddot{\ddot{\ddot{x}}}_0y_0 - \ddot{\ddot{\ddot{y}}}_0x_0)\tau^4 + \dots \end{aligned}$$

Introduce:

$$\frac{1}{r^3} = u \quad f(\mathfrak{M}_1 + \mathfrak{M}_2)p^3 = k^2$$

(In the last expression the designations used by Zagar have been changed in order to obtain more suitable units.)

Then the equations of motion lead to the following expressions of $\ddot{x}_0$, $\ddot{\ddot{x}}_0$ and $\ddot{\ddot{\ddot{x}}}_0$:

$$(24) \quad \begin{aligned} \ddot{x}_0 &= -k^2u_0x_0 \\ \ddot{\ddot{x}}_0 &= -k^2(\dot{u}_0x_0 + u_0\dot{x}_0) \\ \ddot{\ddot{\ddot{x}}}_0 &= -k^2[(\ddot{u}_0 - k^2u_0^2)x_0 + 2\dot{u}_0\dot{x}_0] \end{aligned}$$

and to analogous expressions of $\ddot{y}_0$, $\ddot{\ddot{y}}_0$ and $\ddot{\ddot{\ddot{y}}}_0$. These expressions are introduced in (23), together with the expressions of the double areal constant for the time $\tau=0$:

²¹ Methode zur Bahnbestimmung der Doppelsterne. AN **124**, col. 215 (1890).

²² Orbita del sistema binario Σ 1909 (44 i Bootis). Atti del R Istituto Veneto **97**:2, p. 75 (1937) = Padova Pubbl **57** (1938).

Nuove ricerche sull'orbita del sistema binario 44 i Bootis. Atti del R Istituto Veneto **99**:2, p. 115 (1940) = Padova Pubbl **66** (1940).

²³ Sopra un nuovo metodo analitico per il calcolo d'orbita delle doppie visuali. Atti della R Accademia di Palermo **1** (1938) = Pubblicazioni dell'Osservatorio Astronomico di Palermo **8**:6 (1938).

$$(25) \quad x_0 \dot{y}_0 - \dot{x}_0 y_0 = c$$

After reduction the following relation is obtained:

$$(26) \quad \alpha x + \beta y + \gamma \tau^3 + \delta \tau^4 = \tau$$

where

$$(27) \quad \alpha = -\frac{y_0}{c} \quad \beta = +\frac{x_0}{c} \quad \gamma = +\frac{1}{6} k^2 u_0 \quad \delta = +\frac{1}{12} k^2 \ddot{u}_0$$

By multiplying the first of the equations (22) by $\dot{y}_0$, the second by $\dot{x}_0$, we obtain an equation which may be transformed into

$$(28) \quad \alpha' x + \beta' y + \gamma \cdot 3\tau^2 + \delta \cdot 2\tau^3 + \varepsilon \cdot \tau^4 = 1$$

where

$$(29) \quad \alpha' = +\frac{\dot{y}_0}{c} \quad \beta' = -\frac{\dot{x}_0}{c} \quad \varepsilon = \frac{1}{24} k^2 (\ddot{u}_0 - k^2 u_0^2) = \frac{1}{24} k^2 \ddot{u}_0 - \frac{3}{2} \gamma^2$$

Each observation (or normal place) gives one equation of type (26) with the four unknowns α , β , γ , δ , and one of type (28) with the five unknowns α' , β' , γ , δ , ε . In this way two systems of normal equations are obtained. According to Zagar, it is generally advisable to solve these systems separately, instead of combining them with equal weights into one system with seven unknowns. The reason for this will be given later.

When the seven quantities above are known, the values of x_0 , y_0 , z_0 , $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$ and k^2 can next be derived; the signs of z_0 and $\dot{z}_0$ are, however, undetermined. The value of z_0 is obtained from a biquadratic equation.

Finally, the orbital elements are computed according to well-known formulæ.

Zagar also makes some important remarks on the practical use of the method. Equation (23) is formed by the combination of the two equations (22), after multiplying these by y_0 and x_0 , respectively. These equations will thus enter with weights which depend on the mean errors in y_0 and x_0 . The mean errors ought to be equal, and this is likely to be the case if x_0 and y_0 are approximately equal. In the same way it is found that the condition $\dot{x}_0 = \dot{y}_0$ ought to be approximately fulfilled. It follows that the radius vector in the point (x_0, y_0) ought to form an angle of about 45° with the x -axis, and further that the tangent of the arc in the same point ought to be as nearly as possible perpendicular to the radius vector. The epoch $\tau=0$ and the origin of the position angles should thus be chosen with a view to the above conditions.

Even if these conditions are fulfilled, it is clear that the values of x_0 and y_0 on one side and $\dot{x}_0$ and $\dot{y}_0$ on the other will generally not be determined with the same accuracy. This is the reason why the equations of type (26)

should not simply be combined with those of type (28) into one system. The final values of the two unknowns γ and δ , which occur in both systems, will be the weighted means of the values obtained from the separate solutions of the two systems; the weighting is carried out according to the mean errors of γ and δ in each system. — According to Zagar, the following method is still more accurate. The systems are first solved separately, and the mean errors of x_0 , y_0 and $\dot{x}_0$, $\dot{y}_0$ are determined. The two systems are weighted according to these mean errors and are then joined into one system, from which the seven unknowns are obtained.

In the same paper Zagar applies the method to the system of $\Sigma 1909 = 44$ i Bootis = ADS 9494.

It seems that Silva's method, as modified by Zagar, is the most practicable one of those given so far in this section. Zagar states that the method is especially suitable when the apparent orbit is nearly a straight line. The method has, however, one disadvantage, which it shares with the other methods given above: the use of the rectangular co-ordinates instead of the polar co-ordinates measured in micrometer work. That this really is a disadvantage, is due to the nature of the errors in the measurements. The errors in distance and in position angle are of quite different kinds, and in general the former are larger than the latter, as expressed in arc measure. It is, however, a favourable circumstance that the last-mentioned difference is largely smoothed out when the pairs are wide.

The point of view that an orbit determination ought to be founded on the polar co-ordinates and not on the rectangular ones has been emphasized by W. RABE, who has developed a method in accordance with this opinion.²⁴

In Rabe's method two different cases are distinguished: according to the form of the orbit, the calculations may be based either on the position angles or on the distances. In the former case the position angles are first expressed in power series of the time:

$$(30) \quad \theta = \theta_0 + \alpha_1\tau + \alpha_2\tau^2 + \alpha_3\tau^3 + \alpha_4\tau^4 + \alpha_5\tau^5 + \dots$$

Here τ is the time counted from a suitable epoch and measured in a convenient unit so that $|\tau| \leq 1$ for all the observations. By the method of least squares, the values of θ_0 , α_1 , α_2 , α_3 , α_4 , α_5 , . . . can be determined from the observed position angles and the times of observation.

By derivating with respect to τ we obtain:

$$(31) \quad \dot{\theta} = \alpha_1 + 2\alpha_2\tau + 3\alpha_3\tau^2 + 4\alpha_4\tau^3 + 5\alpha_5\tau^4 + \dots$$

²⁴ Über Bahnbestimmung und Bahnverbesserung visueller Doppelsterne aus kurzen Bahnbögen. AN 265, col. 177 (1938).

If we put:

$$(32) \quad \rho^2 = \rho_0^2(1 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 + \dots)$$

we obtain by multiplying (31) and (32):

$$(33) \quad \rho^2\ddot{\theta} = \rho_0^2[\alpha_1 + (\alpha_1a_1 + 2\alpha_2)\tau + (\alpha_1a_2 + 2\alpha_2a_1 + 3\alpha_3)\tau^2 + (\alpha_1a_3 + 2\alpha_2a_2 + 3\alpha_3a_1 + 4\alpha_4)\tau^3 + (\alpha_1a_4 + 2\alpha_2a_3 + 3\alpha_3a_2 + 4\alpha_4a_1 + 5\alpha_5)\tau^4 + \dots]$$

This is the expression of the double areal constant and must thus be independent of τ . The following relations are thus obtained:

$$(34) \quad \begin{aligned} \alpha_1a_1 + 2\alpha_2 &= 0 \\ \alpha_1a_2 + 2\alpha_2a_1 + 3\alpha_3 &= 0 \\ \alpha_1a_3 + 2\alpha_2a_2 + 3\alpha_3a_1 + 4\alpha_4 &= 0 \\ \alpha_1a_4 + 2\alpha_2a_3 + 3\alpha_3a_2 + 4\alpha_4a_1 + 5\alpha_5 &= 0 \\ &\dots\dots\dots \end{aligned}$$

From these relations the values of $a_1, a_2, a_3, a_4, \dots$ are successively computed.

If the determination of the orbit is to be based on the distances, the necessary formulæ can be derived in a similar way.

By derivating the equations (31) and (32) repeatedly and putting $\tau=0$ we obtain the values of $\dot{\theta}_0, \ddot{\theta}_0, \ddot{\theta}_0, \dots$ and $\dot{\rho}_0, \ddot{\rho}_0, \ddot{\rho}_0, \ddot{\rho}_0, \dots$ as functions of $\alpha_1, \alpha_2, \alpha_3, \dots, \rho_0, a_1, a_2, a_3, a_4, \dots$. The rectangular co-ordinates x_0, y_0 and their derivatives can next be expressed as functions of θ_0, ρ_0 and their derivatives. After reductions we obtain the following relations, in which the index 0 has been left out everywhere.

$$(35) \quad \begin{aligned} x &= \rho \cos \theta \\ y &= \rho \sin \theta \\ \dot{x} &= \dot{\rho} \cos \theta - y\dot{\theta} \\ \dot{y} &= \dot{\rho} \sin \theta + x\dot{\theta} \\ \ddot{x} &= \ddot{\rho} \cos \theta - \dot{y}\dot{\theta} - \frac{1}{2}y\ddot{\theta} \\ \ddot{y} &= \ddot{\rho} \sin \theta + \dot{x}\dot{\theta} + \frac{1}{2}x\ddot{\theta} \\ \ddot{x} &= \ddot{\rho} \cos \theta - \ddot{y}\dot{\theta} - \frac{3}{2}x\dot{\theta}\ddot{\theta} \\ \ddot{y} &= \ddot{\rho} \sin \theta + \ddot{x}\dot{\theta} - \frac{3}{2}y\dot{\theta}\ddot{\theta} \\ \ddot{x} &= \ddot{\rho} \cos \theta - \ddot{\rho}\dot{\theta} \sin \theta - \ddot{y}\dot{\theta} - \ddot{y}\ddot{\theta} - \frac{3}{2}\dot{x}\dot{\theta}\ddot{\theta} - \frac{3}{2}x\ddot{\theta}^2 - \frac{3}{2}x\dot{\theta}\ddot{\theta} \\ \ddot{y} &= \ddot{\rho} \sin \theta + \ddot{\rho}\dot{\theta} \cos \theta + \ddot{x}\dot{\theta} + \ddot{x}\ddot{\theta} - \frac{3}{2}\dot{y}\dot{\theta}\ddot{\theta} - \frac{3}{2}y\ddot{\theta}^2 - \frac{3}{2}y\dot{\theta}\ddot{\theta} \end{aligned}$$

The values of k^2 , $\pm z_0$ and $\pm \dot{z}_0$ can now be derived by means of the following set of equations:

eq. (13), (15) and (16), p. 73;

Schwarzschild's equation, written in the form:

$$(36) \quad r^4 \left[\frac{\ddot{r}}{r} - \frac{\ddot{x}}{x} \right] - r^2 \rho^2 \left[\left(\frac{\ddot{r}}{r} - \frac{\ddot{x}}{x} \right) + \left(\frac{\dot{\rho}}{\rho} - \frac{\dot{r}}{r} \right)^2 + \left(\frac{\ddot{\rho}}{\rho} - \frac{\ddot{x}}{x} \right) \right] + \rho^4 \left[\frac{\ddot{\rho}}{\rho} - \frac{\ddot{x}}{x} \right] = 0$$

and the two equations

$$(37) \quad z^2 = r^2 - \rho^2 \quad z\dot{z} = r\dot{r} - \rho\dot{\rho}$$

(In the equations (36) and (37) the index 0 has been left out.)

Since the values of k^2 , x_0 , y_0 , $\pm z_0$, $\dot{x}_0$, $\dot{y}_0$, $\pm \dot{z}_0$ are now known, the orbit elements can be computed from known formulæ.

As Rabe points out, the above process is, however, seldom practicable, since it implies the knowledge of the derivatives $\ddot{x}$ and $\ddot{y}$, which can hardly ever be determined accurately enough. Rabe therefore derives a value of k^2 from the parallax and the total mass of the system, in the same way as was done before by Fletcher. The solution then becomes considerably simpler, since the equations (16) and (36) are no longer necessary.

The elements obtained in this way are of a preliminary character. To improve them, Rabe has worked out a method which can be used even if the preliminary elements have been obtained according to another method; the method is also useful if new observations are to be included in the computation. The first step is to assign a change to the value of $\frac{\dot{r}_0}{r_0}$ in such a way that the values of z_0 , $\dot{x}_0$, $\dot{y}_0$ and $\dot{z}_0$ are changed but k^2 , x_0 and y_0 remain unchanged. This process is called »variation of $\frac{\dot{r}_0}{r_0}$ of the first kind». The elements are recomputed with the new values, and by comparing the representation of selected normal places in the two cases it is possible to find a better value of $\frac{\dot{r}_0}{r_0}$. A further improvement is obtained by a new variation of the value of $\frac{\dot{r}_0}{r_0}$ (»variation of $\frac{\dot{r}_0}{r_0}$ of the second kind»), by which $\dot{z}_0$ is changed, while the other quantities are unchanged. At last, r_0 is subjected to a variation which affects the values of k^2 , z_0 and $\dot{z}_0$. It may be pointed out that by this last variation the computation is relieved of the dependence on the parallax and the mass of the system.

In Rabe's paper the method is applied to the systems of Castor=ADS 6175 and Σ 3062=ADS 61.

The method has further been successfully applied by W. v. BEZOLD²⁵ to the systems of $\Sigma 1536 = \iota$ Leonis = ADS 8148, $\Sigma 1819 =$ ADS 9182 and $\Sigma 2130 = \mu$ Draconis = ADS 10345; and by F. SCHMEIDLER²⁶ to the system of $\Sigma 1687 = 35$ Comae = ADS 8695.

RABE²⁷ has later extended his method of improving the orbits, so that it may be used even if the measured arc is long.

The methods given by Rabe are, no doubt, of great value. It is true that they involve a great deal of computation work, but Rabe has shown that the process of the orbit improvement may be simplified, without affecting the accuracy to any considerable degree.

A new method of computing the orbit elements when the coefficients of the apparent ellipse are known has been given by T. RAKOWIECKI.²⁸ The determination is carried out by way of the elements of the apparent orbit.

E. MARTIN²⁹ has given a synopsis of the analytical methods of J. Herschel, Villarceau, Kowalski and Seeliger. He shows that these methods, as well as several others, are founded on the same principle. In fact, they all start from the equation of the apparent ellipse, and the five elements e , a , i , Ω and ω are derived from the five coefficients in this equation. The differences between the methods thus refer only to the procedure of computation. — Martin also deduces a direct relation between the measured quantities θ and ρ and the five elements mentioned above. This relation may be written as follows:

$$(38) \quad \rho \sqrt{\cos^2 (\theta - \Omega) + \sin^2 (\theta - \Omega) \sec^2 i} = a(1 - e^2) - e \cos \omega \cdot \rho \cos (\theta - \Omega) - e \sin \omega \sec i \cdot \rho \sin (\theta - \Omega)$$

The squares of both members are taken, and since the relation must be identically valid for all values of θ and ρ , it leads to five equations. These express the coefficients of the equation of the apparent ellipse directly as functions of the above-mentioned five elements.

²⁵ Die Bahnen der Doppelsterne $\Sigma 1536$, $\Sigma 1819$ und $\Sigma 2130$ nach der Methode von W. Rabe. AN **267**, col. 221 (1938).

²⁶ Provisorische Bahnbestimmung des Doppelsternsystems $\Sigma 1687 = 35$ Comae. AN **268**, col. 151 (1939).

²⁷ Zur Bahnverbesserung visueller Doppelsterne nach der Methode der Variation des Radiusvektors und seiner Geschwindigkeit bei Bahnbögen beliebiger Grösse. AN **268**, col. 1 (1939).

²⁸ Nouvelle méthode pour la détermination des orbites des étoiles doubles télescopiques. Prace Mat-Fiz **42**, p. 171 (1934) = Wars Publ **9**, p. 39 (1934).

²⁹ Su alcuni metodi analitici per il calcolo d'orbita di una binaria visuale. Atti del R Istituto Veneto **92**:2, p. 171 (1932) = Padova Pubbl **29** (1933). — Abstract in English: On some analytical methods for calculating the orbit of a visual binary star. R Stazione Astronomica e Geofisica di Carloforte.

The method of the hodographic ellipse.

E. DE CARO³⁰ has developed an orbit determination method which is, from a practical point of view, probably not superior to those treated above. Owing to its great theoretical interest it will, however, be briefly reviewed below.

The method is founded on the use of the hodograph, or the curve described by the end-point of the radius vector that represents the velocity of the companion.

We assume that the orbit is an ellipse, and that the equation of the apparent orbit is given by eq. (17), p. 74. Then it can be shown that the hodograph in the plane of the true orbit is a circle with radius

$$(39) \quad R = \frac{k}{\sqrt{a(1-e^2)}}$$

Of course, the centre of this circle does generally not coincide with the origin. — The projection of the circle on the celestial plane is an ellipse, the hodographic ellipse; its equation may be written:

$$(40) \quad A_1 \dot{x}^2 + B_1 \dot{y}^2 + 2C_1 \dot{x}\dot{y} + 2D_1 \dot{x} + 2E_1 \dot{y} = 1$$

As de Caro demonstrates, the quantities Ω , e , i , ω and R can be obtained from very simple geometrical considerations if the hodographic ellipse has been drawn.

The analytical solution is performed as follows:

The values of x and y are known for different epochs. The corresponding values of $\dot{x}$ and $\dot{y}$ are obtained by expanding x and y in power series of the time and continuing in the same way as in Zagar's method.³¹

By derivating the equation of the apparent orbit with respect to x and using the relation $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ the following equation can be obtained:

$$(41) \quad \frac{A}{E} x \dot{x} + \frac{B}{E} y \dot{y} + \frac{C}{E} (x \dot{y} + y \dot{x}) + \frac{D}{E} \dot{x} = -\dot{y}$$

In a similar way, eq. (40) leads to the following equation:

$$(42) \quad \frac{A_1}{E_1} x \dot{x} + \frac{B_1}{E_1} y \dot{y} + \frac{C_1}{E_1} (x \dot{y} + y \dot{x}) + \frac{D_1}{E_1} \dot{x} = -\dot{y}$$

Each observation (or normal place) furnishes one equation of type (41) and one of type (42). De Caro rewrites the equations in a somewhat modified

³⁰ Sul moto orbitale di un sistema binario visuale. Mem SAI (NS) **12**, p. 27 (1939)
= R Oss Astr Vincenzo Cerulli Teramo, Note e Comunicazioni **23** (1939).

³¹ See p. 75 in this paper.

way; they are then subjected to a least squares solution, which gives the values of $\frac{A}{E}, \frac{B}{E}, \frac{C}{E}, \frac{D}{E}$ and $\frac{A_1}{E_1}, \frac{B_1}{E_1}, \frac{C_1}{E_1}, \frac{D_1}{E_1}$. The values of $\frac{1}{E}$ and $\frac{1}{E_1}$ can now be derived from (17) and (40).

The coefficients A, B, C, D, E and A_1, B_1, C_1, D_1, E_1 may thus be regarded as known. From the former set of coefficients Ω, e, i, ω and the half parameter $a(1-e^2)$ can be computed from known formulæ. The latter set of coefficients affords, as shown by de Caro, the possibility of computing Ω, e, i, ω and $R = \frac{k}{\sqrt{a(1-e^2)}}$. The circumstance that some elements are obtained in two ways will furnish a control of the computations. — Finally it is shown how the values of P and T can also be computed.

In the same paper the method is applied to Sirius (AB)=ADS 5423.

Some graphical methods.

E. MARTIN has published two papers dealing with graphical methods of computing binary orbits. The first paper³² gives a summary of different principles for the determination of the position angle of the node (Ω), together with a new method of determining this element. This method has the advantage that several independent values of Ω can be obtained, which will of course increase the reliability of the determination. It also gives an indication of the degree of accuracy attained. — When Ω has been found, the values of e, i, ω , and a are easily determined.

In the second paper³³ Martin first points out that the elements should, as far as possible, be determined independently of each other. In fact, if one element is used for the determination of another, then an error in the former element will give rise to an error in the latter. Assuming that the apparent ellipse has been drawn, Martin develops a graphical method in which e, i and ω are determined independently of a and Ω .

A. L. ZELMANOW³⁴ has given a method which is similar to Zwiers'; only a few changes in the method of deriving the elements have been made.

³² Metodo (I°) per il calcolo d'orbita di una binaria visuale. *Lincei Rend* (6) **15**, p. 955 (1932) = *Padova Pubbl* **24** (1932). — Abstract in English: Method for calculating the orbit of a visual binary star. *R Stazione Astronomica e Geofisica di Carloforte*.

³³ Metodo II° per il calcolo d'orbita di una binaria visuale. *Lincei Rend* (6) **16**, p. 231 (1932) = *Padova Pubbl* **25** (1932).

³⁴ A Method for the Computation of the Orbits of Visual Binaries. *Astr Journal of Soviet Union* **16**:4, p. 25 (1939).

N. W. McLEOD³⁵ has shown that the following device makes it possible to determine the period of a binary without using a planimeter, even if the companion has not completed a whole revolution.

It is assumed that the apparent ellipse has been drawn. The ends of the projected parameter passing through the primary are first determined. This may easily be done geometrically, since the projected parameter is parallel to the projected minor axis, which is the conjugate diameter of the projected major axis. It is also easily done algebraically by means of formulæ given in McLeod's paper.

The times when the companion passed the ends of the parameter are then read off from the interpolation curve that gives the position angle as a function of the time. The difference between these times gives either the time of passing through the periastron part of the orbit (x_p), or that of passing through the apastron part (x_a). The period is then found from one of the two following formulæ:³⁶

$$(43) \quad \begin{aligned} P &= \frac{2\pi}{\pi - 2e} \cdot x_p \\ P &= \frac{2\pi}{\pi + 2e} \cdot x_a \end{aligned}$$

W. SCHAUB³⁷ has proposed a method in which the rectangular co-ordinates x and y are plotted as functions of the time instead of the polar co-ordinates θ and ρ . Interpolation curves are drawn, and adjustments are made so as to make the resulting apparent orbit satisfy the law of areas and the condition of elliptical path. Some additional conditions that must be satisfied by the curves are also given.

Schaub also makes a theoretical investigation of the form of the expressions for the rectangular co-ordinates.

The method is applied to κ Hydri, which is suspected to be a double star, and to α Centauri=CPD $-60^\circ 5483$.³⁸

It may be remarked that the method of basing the adjustment on the (x,t) - and the (y,t) -curves hardly seems to be simpler than the usual one, in which the (θ,t) - and the (ρ,t) -curves are used. It seems suitable to use the former method when the orbit determination is based on meridian observations (as in the case of κ Hydri) or on photographic measurements. If, how-

³⁵ A Method of finding the Period of a Binary Star when the Geometric Elements are Known from Zwiers' Method. AJ 48, p. 119 (1939).

³⁶ The formulæ are given in a form which is somewhat different from that used by McLeod.

³⁷ Die Eigenbewegung des Sternes κ Hydri, FK3Z Nr 1067. AN 272, p. 182 (1942). Grundlagen und Beispiel für die Ableitung der scheinbaren Bahn eines Doppelsternes aus den relativen rechtwinkligen Koordinaten. AN 272, p. 185 (1942).

³⁸ The latter determination is based only on observations up to 1907.

ever, the material consists of micrometer measurements, there is little doubt that it is more correct to found the adjustment on the polar co-ordinates.

For the sake of completeness, references is given below to a short article by R. LAUFFER³⁹ which has not been accessible.

Danjon's method (and a few other methods).

The method of orbit computation proposed by A. DANJON⁴⁰ is partly based on new and very interesting ideas. It utilizes the properties of *opposite points*, or points whose position angles differ by 180° . If two such points are indicated by the indices 1 and 2, it can be shown that the function

$$(44) \quad \Delta = M_2 - M_1 - 180^\circ$$

(where M_1 and M_2 are counted in degrees) presents several noticeable features. If Δ is regarded as a function of M_1 , it is found to be zero for $M_1=0^\circ$ (periastron); positive from periastron to apastron; zero for $M_1=180^\circ$ (apastron); negative from apastron to periastron. Further, if Δ is represented graphically as a function of M_1 , it is found that the points $(M_1=0^\circ, \Delta=0^\circ)$ and $(M_1=180^\circ, \Delta=0^\circ)$ are centres of symmetry. Thus it is sufficient to study the interval $0^\circ \leq M_1 \leq 180^\circ$.

These curves, which Danjon calls »*characteristic curves*» (»*courbes caractéristiques*»), have the important quality that their form depends only on the eccentricity. Thus it is possible to compute the theoretical curves for different values of e . Danjon recommends that these curve should be drawn on transparent paper, using a large scale.

In the case when the period P is known from recurrence of angle, approximate values of the time of periastron passage (T) and the eccentricity can be determined by comparing these standard curves with the corresponding curve that is obtained from the measured position angles. The latter curve is obtained with the help of the interpolation curve giving the dependence of the position angles on the time, using several pairs of opposite points. — Danjon shows how the approximate values of T and e can be corrected analytically.

Danjon further shows that the method is valid even if the period is not known from the beginning.

The next object is to express three of the Thiele-Innes constants as

³⁹ Zur Konstruktion der wahren Doppelsternbahn. Unterrichtsblatt für Mathematik und Naturwissenschaft **47**, p. 195 (1941).

⁴⁰ Nouvelle méthode pour la détermination des éléments des étoiles doubles visuelles. BA Mémoires et Variétés **11**, p. 191 (1938). — Summary: Détermination des éléments des orbites des étoiles doubles visuelles. CR **206**, p. 322 (1938).

functions of the fourth. We start from the following relation between the position angle, the true anomaly and the Thiele-Innes constants. (The relation is written for one of two opposite points; this is indicated by the index 1.)

$$(45) \quad \operatorname{tg} \theta_1 = \frac{B + G \operatorname{tg} v_1}{A + F \operatorname{tg} v_1}$$

The value of v_1 that corresponds to a given value of θ_1 is computed as follows:

1) $M_2 - M_1$ can be obtained from the (θ, t) -curve.

2) $E_2 - E_1$ is obtained from the following relation, which is Thiele's fundamental equation in the special case of opposite points:

$$(46) \quad M_2 - M_1 = E_2 - E_1 - \sin(E_2 - E_1)$$

3) The value of v_1 is computed from the following relation, which is valid for opposite points:

$$(47) \quad \sin v_1 = -\frac{\sqrt{1-e^2}}{e} \cot \frac{1}{2}(E_2 - E_1)$$

The computations are facilitated by special tables given in Danjon's paper.

We can thus assume that several pairs of values of $\operatorname{tg} \theta_1$ and $\operatorname{tg} v_1$ are known. The following designations are now introduced:

$$(48) \quad x = \operatorname{tg} \theta_1 \quad y = \operatorname{tg} v_1 \quad \alpha = \frac{G}{F} \quad \beta = -\frac{A}{F} \quad \gamma = \frac{BF - AG}{F^2}$$

Eq. (45) may then be written:

$$(49) \quad (x - \alpha)(y - \beta) = \gamma$$

The values of α , β and γ can be determined according to the method of least squares, but Danjon prefers the following procedure.

If $\theta_1 = 90^\circ$ or 270° , i.e. $x = \pm \infty$, then $y = \beta$. This leads to an approximate value β' of β .

The equation is then written as follows:

$$(50) \quad x(y - \beta') = \gamma + \alpha(\beta - \beta') + \alpha(y - \beta) + \frac{\gamma(\beta - \beta')}{y - \beta}$$

If the value of $y - \beta$ is not too small, the last term may be neglected, and then the product $x(y - \beta')$ must be a linear function of y . The graphical representation of this product as a function of y must thus be a straight line, and the angle coefficient of this line gives the value of α .

An improved value of β can be obtained in a similar way, and finally the mean value of $(x - \alpha)(y - \beta)$ gives γ .

All the previous calculations rest upon the angles alone, and this is justified by the fact that the method concerns close pairs, for which the measurements of the position angles are more accurate than those of the distances. The constant F must, however, now be determined by the aid of the distances. This is done by computing an ephemeris of the distances, using the value $F=1$, and plotting the computed distances against the observed ones. The points ought to lie on a straight line, whose angle coefficient gives the value of F . According to Danjon, this manner of proceeding is to be preferred to the least squares method, since it gives the opportunity of taking the systematic errors of different observers into consideration.

Since the constants A , B , F and G are now known, the elements a , i , Ω and ω can be computed.

As illustrative examples, the paper contains the determination of the orbits of η Coronae=ADS 9617 and φ Ursae Majoris=ADS 7545.

Danjon's method is no doubt of great value from both a practical and a theoretical point of view. Its range of applicability is, however, restricted by the fact that several pairs of opposite points are needed, and thus a large part of the orbit must be known.

The idea of using the opposite points is not new; Danjon mentions that their properties have been studied by G. SCHNAUDER.⁴¹ The opposite points have also been utilized by T. RAKOWIECKI in two papers which are mentioned by Danjon. In the former investigation⁴² the distance measurements of the opposite points are also used. The second investigation⁴³ makes use of only two pairs of opposite points. Rakowiecki's methods are thus different from those used by Danjon, and they are probably of less importance from a practical point of view.

The two above-mentioned papers by Rakowiecki have not been accessible, and the same is the case with a third paper by Rakowiecki⁴⁴ and one paper by P. BAIZE;⁴⁵ references to these papers are given at the bottom of the page.

⁴¹ Über abzählende Methoden der Bahnbestimmung von Doppelsternen. AN **216**, col. 129 (1922).

⁴² Deux méthodes pour la détermination de l'orbite réelle du satellite de l'étoile double visuelle à l'aide des positions opposées. Wiad Mat **42**, p. 85 (1936).

⁴³ Détermination de l'orbite du compagnon de l'étoile double visuelle à l'aide des temps et des angles de position, Prace Mat-Fiz **46**, p. 245 (1938).

⁴⁴ Détermination des éléments de l'orbite apparente du compagnon d'une binaire visuelle par interpolation des positions observées. Wiad Mat **46**, p. 125 (1938).

⁴⁵ Méthode simple pour calculer l'orbite d'une étoile double. Applications à Σ 3062. BSAF **55**, p. 203 (1941).

Differential corrections.

When a preliminary set of elements of a double star orbit has been derived, it is often found that the material is sufficient for the improvement of the elements by differential corrections. If such corrections are to be applied, the following two conditions ought to be fulfilled: 1) The corrections must be so small that their second and higher powers may be neglected; 2) A correction to one element must not affect the others seriously.

The computation of differential corrections is treated by Aitken in *The Binary Stars*.⁴⁶ He first discusses the case that corrections to Campbell elements are to be based on the polar co-ordinates. The differential coefficients (or the partial derivatives of the co-ordinates with respect to the elements) are given in the form derived by G. C. COMSTOCK.⁴⁷ Each measurement of angle or distance furnishes one equation. In fact, each residual is equal to the sum of six terms, which are the products of the desired corrections and the corresponding differential coefficients. The equations form a system, which is solved by the least squares method. — The case that the Thiele-Innes elements and the rectangular co-ordinates are used, is also treated, according to the method given by W. H. VAN DEN BOS.⁴⁸ In this case each equation contains the corrections to five elements as unknowns. These equations are also intended to be treated according to the method of least squares.

A modification of the last method has been given by W. H. VAN DEN BOS.⁴⁹ He only makes use of the seven equations that are necessary for the solution of the seven differential corrections. These equations can be obtained from three normal places together with a fourth normal place used in one co-ordinate. However, van den Bos prefers to use the double areal constant instead of the fourth normal place. The expression of the double areal constant c may be written:

$$(51) \quad 57.3 \, c = n \cos \varphi (AG - BF)$$

where $\sin \varphi = e$. By differentiating this equation we have:

$$(52) \quad 57.3 \, \Delta c = n \cos \varphi (G\Delta A - F\Delta B - B\Delta F + A\Delta G) - n \operatorname{tg} \varphi (AG - BF) \Delta e + \\ + \cos \varphi (AG - BF) \Delta n$$

Like other methods for differential corrections, this one is also available if it is desired to include new observations together with those used in the

⁴⁶ P. 109—113.

⁴⁷ The Orbit of Σ 2026 = B 7561. AJ **31**, p. 33 (1918).

⁴⁸ See references on p. 68 in this paper.

⁴⁹ Correction of a Double Star Orbit; Orbit of β 744. UOC **90**, p. 393 (1933).

original orbit determination. In this case it is recommendable to select two normal places from the original observations and one from the new ones.

In the same paper the method is applied to the orbit of β 744=ADS 3159.

It has already been mentioned ⁵⁰ that VAN DEN BOS is of opinion that the method of least squares is often not suitable for the computation of double star elements. In an important paper ⁵¹ he states that there are at least two objections to this method: firstly, that the results obtained often do not justify the rather large computation work; secondly, that the character of the errors of measurement is such that the least squares method is often not likely to afford the most reliable solution.

For that reason van den Bos has worked out a method which involves less computation work than the least squares method, at the same time leaving more room for the computer's judgement. — Two different cases are distinguished: if the orbit is a fairly open ellipse, the adjustment is to be based on the angles; if, on the other hand, the orbit is very narrow, then the distances are of more importance than the angles. The two cases will be treated below.

1) *The adjustment is to be based on the angles.*

The fundamental equation may then be written:

$$(53) \quad \Delta\theta = \Delta\Omega + \frac{\partial\theta}{\partial i} \Delta i + \frac{\partial\theta}{\partial \omega} \Delta\omega + \frac{\partial\theta}{\partial e} \Delta e + \frac{\partial\theta}{\partial \tau} \Delta\tau + \frac{\partial\theta}{\partial n} \Delta n$$

where $\Delta\tau = -n\Delta T$. — Formulæ for the computation of the differential coefficients $\frac{\partial\theta}{\partial i}, \frac{\partial\theta}{\partial \omega}, \dots$ are given in van den Bos' paper.

The values of each set of differential coefficients are next plotted against the times, the mean anomalies, or the serial numbers of the observations. The residuals $\Delta\theta$ are plotted in the same way on tracing-paper. The points of each plot are joined by straight lines. Convenient scales should be used, so that the amplitudes of the different curves are comparable to each other.

The plot on the tracing-paper is now compared with the other plots by placing it on them, either face up or face down, so that the abscissæ agree. A vertical shift of the curves is allowed, since it is equivalent to an error which may be corrected by changing the value of Ω ; this element is corrected after the five others have been treated.

The element whose coefficient curve shows the nearest resemblance to the residual curve is first corrected. If the tracing-paper has been placed face up, the correction ought to be positive; if it has been placed face down,

⁵⁰ See p. 74, Note 16.

⁵¹ Differential Correction of the Orbit of a Visual Binary. UOC 98, p. 337 (1937).

the correction is negative. The size of the correction is estimated, and the corresponding corrections are applied to the residuals. The new $\Delta\theta$ -curve is compared with the coefficient curve, and if necessary a further adjustment is made. — The process is continued in the same way for the other elements, and finally the adjustment of Ω is made. — An ephemeris of both position angles and distances is then computed, using the provisional value $\alpha=1''$. The residuals in angle should agree with those which are obtained when all the adjustments have been made. From the most reliable distance measurements, as compared with the computed provisional distances, the value of α is determined.

In the paper this method is illustrated on $\text{O}\Sigma\ 79=\text{ADS}\ 3135$.

2) *The adjustment is to be based mainly on the distances.*

In this case, when the apparent orbit is a very narrow one, an axis, x' , is laid parallel to the major axis of the apparent ellipse, and another axis, y' , perpendicular to it. If the corresponding values of the Thiele-Innes constants are A' , B' , F' , G' , the following equation for $\Delta x'$ is obtained:

$$(54) \quad \Delta x' = \frac{\partial x'}{\partial A'} \Delta A' + \frac{\partial x'}{\partial F'} \Delta F' + \frac{\partial x'}{\partial e} \Delta e + \frac{\partial x'}{\partial \tau} \Delta \tau + \frac{\partial x'}{\partial n} \Delta n$$

Since y' is hardly of any value for the correction of e , T and n , the expression of $\Delta y'$ may be written:

$$(55) \quad \Delta y' = \frac{\partial y'}{\partial B'} \Delta B' + \frac{\partial y'}{\partial G'} \Delta G'$$

In (54) and (55) we have $\frac{\partial x'}{\partial A'} = \frac{\partial y'}{\partial B'} = X$, $\frac{\partial x'}{\partial F'} = \frac{\partial y'}{\partial G'} = Y$; the expressions of the other coefficients, which are also simple, are given in the paper.

In the same way as in the first case, eq. (54) is used for deriving the corrections of A' , F' , e , τ and n , and eq. (55) for the corrections of B' and G' . (A vertical shift of the curve is, however, not permitted here.)

The above method of differential corrections has been tested with good results on several orbits given in the *Union Observatory Circulars*.

The method just treated here has the great advantage that the calculations are very simple. On the other hand, it is often difficult to judge of the sizes of the corrections, and thus many a computer will prefer a more objective method. Such considerations have induced W. P. HIRST⁵² to propose a procedure which may, in one modification, be said to be an arithmetical version of the preceding one.

Hirst's paper concerns only the case of a fairly open orbit [case 1) above].

⁵² A Method of applying Differential Corrections to the Elements of Double Star Orbits. MN **103**, p. 337 (1944).

When explaining the fundamental idea of the method, he takes the element e as an example. The value of $\frac{\partial\theta}{\partial e}$ is zero at periastron and apastron; if the motion is direct,⁵³ $\frac{\partial\theta}{\partial e}$ is positive from periastron to apastron, in the direction of motion, and negative from apastron to periastron. If the observations are well distributed with respect to the apsidal line, and a correction to e is not seriously affected by the corrections to the other elements, then the following equation ought to give a good value of the correction Δe :

$$(56) \quad [M_{pa}(e) - M_{ap}(e)] \Delta e = M_{pa}(R) - M_{ap}(R)$$

Here $M_{pa}(R)$ is the mean of the weighted residuals from periastron to apastron and $M_{ap}(R)$ the same function from apastron to periastron. $M_{pa}(e)$ and $M_{ap}(e)$ are the corresponding functions of $\frac{\partial\theta}{\partial e}$.⁵⁴

The elements P , T , i and ω can be corrected in a similar way, if the observed interval is conveniently divided. The following summary shows how to treat the different elements.

For correction of e : the line of apsides is the division line.

» » » P : the observed arc is divided into two about equal parts.

For correction of T : one group is formed by the part of the orbit containing the small distances, the other by that containing the large distances.

For correction of i : take together the opposite quadrants Ω to $\Omega + 90^\circ$ and $\Omega + 180^\circ$ to $\Omega + 270^\circ$.

For correction of ω : take together the opposite quadrants $\Omega + 45^\circ$ to $\Omega + 135^\circ$ and $\Omega + 225^\circ$ to $\Omega + 315^\circ$.

For correction of Ω : no division; the correction is made by using the mean of the adjusted residuals.

The adjustments of the different elements may be made successively, in a way similar to that proposed by van den Bos.

The fact that one element is influenced by errors in the others may be taken into account by the following more accurate method (granted of course that the material justifies its use).

Taking the influence of the other elements into consideration, eq. (56) may be more exactly written as follows:

⁵³ This condition, which is not mentioned by Hirst, is essential; if the motion is retrograde, the opposite signs are to be applied.

⁵⁴ It may be pointed out that it is not quite necessary to make the division strictly according to the apsidal line. The main point is that the difference $M_{pa}(e) - M_{ap}(e)$ must be sufficiently large.

$$(57) \quad [M_{pa}(e) - M_{ap}(e)] \Delta e + [M_{pa}(P) - M_{ap}(P)] \Delta P + [M_{pa}(T) - M_{ap}(T)] \Delta T + \\ + [M_{pa}(i) - M_{ap}(i)] \Delta i + [M_{pa}(\omega) - M_{ap}(\omega)] \Delta \omega = M_{pa}(R) - M_{ap}(R)$$

Similarly, four other equations (using other division points) are obtained for the elements P , T , i and ω . The solution of the system of five equations leads to the desired values of the five corrections.

This last modification will, of course, require considerably more computing work than the previous one, but the work is certainly much less than in the method of least squares.

In any case the expressions of the differential coefficients will be needed. Since the formulæ given by Hirst seem to be preferable to those given in earlier papers, they will be quoted below, with a couple of additions.⁵⁵

$$(58) \quad \begin{aligned} S &= AG - BF = a^2 \cos i \\ R &= n \cos \varphi, \text{ where } \sin \varphi = e \\ \frac{\partial \theta}{\partial T} &= -\frac{RS}{\rho^2} \\ \frac{\partial \theta}{\partial P} &= \frac{t - T}{P} \cdot \frac{\partial \theta}{\partial T} \left[= \frac{M}{360^\circ} \cdot \frac{\partial \theta}{\partial T} \text{ where } M \text{ is always counted} \right. \\ &\quad \left. \text{from the same epoch } T. \right] \\ \frac{\partial \theta}{\partial e} &= -\frac{57.3}{R} \left(X \frac{\partial Y}{\partial e} - Y \frac{\partial X}{\partial e} \right) \cdot \frac{\partial \theta}{\partial T} \\ \frac{\partial \theta}{\partial i} &= -\frac{1}{2} \operatorname{tg} i \sin 2(\theta - \Omega) \\ \frac{\partial \theta}{\partial \omega} &= \cos i + \sin i \operatorname{tg} i \sin^2 (\theta - \Omega) \end{aligned}$$

The above method, in its second modification, is applied by Hirst to the orbit of A 417=ADS 16497.

On the accuracy of orbit determinations.

E. DE CARO⁵⁶ has discussed the errors in the determination of the coefficients of the apparent ellipse. He starts from the equation of this curve, referring to a point (x_i, y_i) on the ellipse:

$$(59) \quad F_i(x, y) = Ax_i^2 + By_i^2 + 2Cxy_i + 2Dx_i + 2Ey_i - 1 = 0$$

⁵⁵ In Hirst's paper the *minus* sign of $\frac{d\theta}{de}$ has been omitted through an oversight, but in his following computations the signs are correct.

⁵⁶ Sul calcolo dei coefficienti geometrici dell'orbita apparente di un sistema binario visuale. Mem SAI (NS) **10**, p. 369 (1937) = R Oss Astr Vincenzo Cerulli Teramo, Note e Comunicazioni **19** (1937).

If, on the other hand, (x_i, y_i) is a point which represents an observation, this point will generally not lie on the ellipse, and the right member is thus generally not zero. If the right member is v_i , we have by differentiation:

$$(60) \quad \left(\frac{\partial F}{\partial x}\right)_i \Delta x_i + \left(\frac{\partial F}{\partial y}\right)_i \Delta y_i = v_i$$

where Δx_i and Δy_i are the errors of measurement in the directions x and y , respectively.

As de Caro shows, the length of the normal drawn from the point (x_i, y_i) to the curve is very nearly equal to

$$(61) \quad \frac{v_i}{\sqrt{\left(\frac{\partial F}{\partial x}\right)_i^2 + \left(\frac{\partial F}{\partial y}\right)_i^2}}$$

It is plausible to determine the constants of the apparent ellipse in such a way that the sum of the squares of these normals will be a minimum. This will be practically equivalent to the condition

$$(62) \quad \sum \frac{v_i^2}{\left(\frac{\partial F}{\partial x}\right)_i^2 + \left(\frac{\partial F}{\partial y}\right)_i^2} = \text{minimum}$$

It is also investigated under which circumstances the fulfilment of this condition may really be expected to lead to the best possible geometrical representation of the observations. It is found that the mean errors of x and y must be equal, and for this it is necessary that the mean errors of the position angles (reduced to arc) are equal to those of the distances.

The case when this last condition is not fulfilled is also treated.

At last it is shown how the above considerations can be utilized for deriving corrections to the coefficients $A, \dots E$.

A very thorough investigation of the accuracy of double star orbit determinations has been made by G. VON SCHRUTKA-RECHTENSTAMM.⁵⁷ Both visual and spectroscopic binaries are treated; the following review applies only to the former class.

The following three assumptions are first made:

- 1) The period is known exactly.
- 2) The sums of the weights of the observations during equal intervals are equal.
- 3) The mean errors of the position angles (reduced to arc) and the distances are equal between themselves and constant for the whole orbit.

⁵⁷ Über die Genauigkeit von definitiven Bahnbestimmungen von Doppelsternen. AN Erg.-Hefte Band 10, Nr 4 (1941) (Diss.) = Mitt der Wiener Sternwarte 2:9, p. 40 (1941).

Under these conditions a general calculation of the accuracy of the orbit elements is carried through. The leading principle is that integrals are introduced instead of the sums that occur in the normal equations in the least squares method.

Several special cases are also treated.

The calculations also furnish the values of the correlation coefficients between the different elements.

The method is illustrated on α Centauri = CPD $-60^{\circ}5483$. It is found that the theoretical errors as computed from the general formulæ are generally smaller than those which are obtained directly from the normal equations. This indicates that the errors of observation do not quite attain the supposed ideal distribution.

From a practical point of view there are two objections to the formulæ given by von Schrutka-Rechtenstamm: they are rather long, and their use is restricted by the three conditions mentioned above. It seems that the formulæ can be used with advantage only for close investigations of particularly well observed binaries. For such investigations, however, they are, no doubt, valuable.

Parabolic orbits.

Several of the methods and the formulæ given earlier in this chapter can be directly applied not only to elliptic orbits, but also to parabolic, or even hyperbolic ones. In other cases the formulæ can easily be changed so that they apply to the two last-mentioned kinds of conics. Of late years there have also been published a few papers that especially deal with the computation of parabolic orbits. Since this question may be considered to be of a more special character, it may be sufficient to give references to a couple of these papers.

In *Union Observatory Circular* **95** W. S. FINSSEN⁵⁸ has extended the Thiele-Innes constants and methods to parabolic orbits. This requires tables of the parabolic rectangular co-ordinates X and Y , which are also given. Parabolic orbits of three double stars are also published in the same *Circular*.

A simple graphical method of determining a parabolic double star orbit has been given by R. v. D. R. WOOLLEY.⁵⁹ Assuming that the apparent orbit has been drawn, Woolley first shows how to construct the projection

⁵⁸ Parabolic Orbits of Double Stars. UOC **95**, p. 225 (1936).

Parabolic Rectangular Co-ordinates. UOC **95**, p. 227 (1936).

Parabolic Orbits of Σ 483, h 3683, BrsO 13. UOC **95**, p. 228 (1936).

⁵⁹ A Parabolic Double-Star Orbit. MN **98**, p. 380 (1938).

of the axis of the true parabola. (The primary must evidently lie on this axis.) The projection of the *latus rectum* can also be constructed. It is also possible to construct the projection of the circle which has its centre in the primary and which passes through the vertex of the parabola. This projection will be an ellipse. The direction of the major axis of this ellipse will be the same as that of the line of nodes, and the ratio of the minor axis to the major axis is equal to the cosine of the inclination. The other orbit elements are now obtained without difficulty.

The method is applied to compute a parabolic orbit of $\Sigma 1639 = \text{ADS } 8539$.

It may often be advantageous to use a parabolic orbit as a substitute for an elliptic one when the eccentricity is nearly $=1$ and the observations do not permit the calculation of an elliptic orbit. However, this does not imply that the orbits should really be parabolic. This has especially been emphasized by N. BONEFF,⁶⁰ who has shown that the probability of the existence of parabolic or hyperbolic systems is extremely small.

⁶⁰ Sur la probabilité de l'existence des systèmes binaires non elliptiques. AN 256, col. 195 (1935).

CHAPTER IV

Ephemeris tables and ephemeris curves for the determination of the orbits of visual binaries

Remarks on the method.

Whatever method we use for the determination of the orbit of a visual double star, it is unavoidable that the derivation of the orbit elements from the observations implies a great deal of work. This holds true, whether the method is analytical or graphical. On the other hand, if the orbit elements are given, it is an easy task to compute an ephemeris which gives the position angle and the distance for known epochs.

Thus it suggests itself to invert the problem of orbit determination. This may be done by computing ephemerides for different values of the orbit elements and then constructing the corresponding ephemeris curves. When these curves are given, it will be comparatively easy to obtain the orbit elements of a certain double star by plotting the observed positions and comparing the diagram with the ephemeris curves.

So far, this method does not seem to have been systematically used for the computation of visual binary orbits. This is probably due to the considerable amount of work that is required for the construction of all the necessary ephemeris curves. It will, however, be of interest to note that an analogous method has been in practice since 1908 for the computation of spectroscopic binary orbits. This method was proposed by W. F. KING.¹ We assume that the period of a spectroscopic binary and the amplitude of the radial velocity curve are known. Then the radial velocity may be represented as a function of the time in such a way that the form of the curve only depends on the eccentricity and the periastron longitude. Preliminary

¹ ApJ 27, p. 125 (1908). An account of the method is also given in *The Binary Stars*, p. 169.

elements are obtained by comparing the curve with a set of previously constructed curves for different values of e^2 and ω . It is suitable to draw these curves on tracing linen. The construction of the curves is considerably facilitated by the fact that, owing to symmetry, it will be sufficient to vary ω from 0° to 90° instead of from 0° to 360° . It is recommended to use the interval 0.05 for e (except for the larger values, which may be taken farther apart) and 15° for ω .

A similar idea was employed by H. N. RUSSELL.³ For the computation of the orbits of spectroscopic binaries he makes use of a set of curves which give the quantity $v-M$ (but for a constant) as a function of M . The form of these curves depends upon e alone.

In this connection the method proposed by A. DANJON⁴ for visual binaries should be remembered. This method, which has been described in Chapter III,⁵ is founded on the »characteristic curves» giving the quantity $\Delta = M_2 - M_1 - 180^\circ$ (where (1) and (2) are opposite points) as a function of M_1 . It has already been mentioned that the form of these curves depends only on e .

K. AA. STRAND⁶ has repeatedly used the following method for the computation of visual orbits. After having derived an approximate set of elements, he computes the ephemerides of orbits with slightly different values of P , e and T . For each orbit the sum of the squares of the weighted residuals is computed, and thus it is possible to find the orbit which makes this sum a minimum. A set of ephemerides is thus computed for each binary whose orbit is to be determined, and the principle of this method is different from that of computing the ephemerides once and for all.

It has already been mentioned that the position angles are generally of greater importance than the distances when the orbit is fairly open, while the contrary is the case when the orbit is narrow. In the continuation only the former case will be treated, and the determination of the elements (except a) is to be founded mainly on the angles. Further it will be assumed that the period of the binary is known, at least approximately. If this is not the case, it is often possible to obtain good results by trying different values of the period.

² The symbols used in this chapter are, if not otherwise stated, the same as in Chapter III (see p. 67—68).

³ ApJ **40**, p. 282 (1914); see also *The Binary Stars*, p. 174.

⁴ See reference on p. 85.

⁵ P. 85—87.

⁶ Leiden Ann **18:2** (1937).

AJ **49**, p. 41 (1940).

The mean anomaly

$$(63) \quad M = \frac{360^\circ}{P} (t - T)$$

is taken as the abscissa of the ephemeris curves, and the quantity

$$(64) \quad \theta_0 = \theta - \Omega$$

as the ordinate.

The co-ordinates of the position angle diagram of the binary whose orbit is to be determined will next be considered. Since the time of periastron passage, T , is unknown, it is obviously not possible to choose M as the abscissa. If the motion is direct we may instead use the quantity

$$(65) \quad M' = \frac{360^\circ}{P} (t - T')$$

where T' is an arbitrarily chosen epoch. From (63) and (65) it is seen that M and M' differ only by a constant. If the motion is retrograde, we may instead use the abscissa

$$(66) \quad M' = -\frac{360^\circ}{P} (t - T')$$

In this case $M + M'$ will be a constant.⁷ — The position angle θ is taken as the abscissa.

In this way a change of T will obviously correspond to a horizontal shift of the diagram, and a change of Ω , to a vertical shift. These elements will thus determine the position of the diagram in relation to the ephemeris curve. The form of the ephemeris curve will depend only on the three elements e , i and ω . It will further be shown below that it is sufficient to vary i and ω from 0° to 90° .

Computation of the tables.

The ephemeris curves, which have been constructed in accordance with the above points of view, are based on tables giving θ_0 as a function of M for different values of e , i and ω . Only values corresponding to $e \leq 0.70$ have been computed, and thus the values of v could be taken directly from the *Allegheny Tables*.⁸ The values of θ_0 have then been computed from the well-known formula

⁷ Another method is to keep the expression (65) of M' ; then the comparison with the ephemeris curves is made by looking through the diagram from the back.

⁸ *Allegheny Publ* 2, p. 155 (1912).

$$(67) \quad \operatorname{tg} \theta_0 = \operatorname{tg}(v + \omega) \cos i \quad (\theta_0 = \theta - \Omega)$$

When the tables were computed, it was found most practical to use e as the first argument. For each value of M , the value of v was taken from the *Allegheny Tables*, and then $v + \omega$ was formed. The value of $\operatorname{tg}(v + \omega)$ was taken from *Lohse's Tafeln*,⁹ which generally give five decimals. By aid of a computing machine this value was multiplied by the different values of $\cos i$, and finally the value of θ_0 was read from *Lohse's Tafeln* to the nearest hundredth of degree. The errors in the computed values will occasionally be larger than $0^\circ.005$, but they should hardly ever exceed $0^\circ.01$. This accuracy will generally be quite sufficient in visual double star work.

The values of θ_0 have been computed for $M=0^\circ, 10^\circ, \dots 360^\circ$. This interval of 10° is generally small enough to permit the accurate construction of the ephemeris curves; only when the curve proved very steep, additional values were interposed.

Tables have been computed for the following different values of e : $0.00; 0.10, \dots 0.70$. For larger values of e , the orbit is generally so narrow that the method will be less suitable.

For i , the following values have been used: $0^\circ, 15^\circ, \dots 75^\circ$. In most cases, however, the method is hardly to be recommended when the inclination is as large as 75° . — Of course, the values for the simple special case $i=90^\circ$ need not be computed. — In the notation adopted by the *I.A.U.*,¹⁰ i is taken in the second quadrant when the motion is retrograde. It has already been indicated that the case of retrograde motion can be treated in the same way as that of direct motion if only a slight modification is made. No tables for $i > 90^\circ$ are thus needed. The elements derived in this case must be changed in order to conform to the notation of the *I.A.U.*

The following values of ω have been used: $0^\circ, 15^\circ, \dots 90^\circ$. It is easily shown that it will be sufficient to compute tables for the case that ω is in the first quadrant. This is seen from formula (67), in which θ_0 must be in the same quadrant as $v + \omega$ if the motion is direct. If ω is changed by 180° , the same will apply to θ_0 . Furthermore, θ_0 will assume the same numerical value, with the opposite sign, if v is changed for $-v$, and ω for $-\omega$. On the other hand, if v corresponds to the mean anomaly M , then $-v$ must correspond to $-M$. Thus the value for $-\omega$ can be obtained by changing the signs of both M and θ_0 . If the curve has been drawn, this will obviously correspond to turning it upside down (face of paper still upwards).

The computation of the tables is simplified in the following *special cases*:

1) $e=0$. The true orbit is a circle, and we may choose $\omega=0$, *i.e.* periastron is taken at the node. Formula (67) is simplified to:

⁹ O. LOHSE, *Tafeln für numerisches Rechnen mit Maschinen*. Leipzig (1st Ed., 1909; 2nd Ed., 1935).

¹⁰ Cf. Note 2 on p. 67.

$$(68) \quad \operatorname{tg} \theta_0 = \operatorname{tg} M \cos i$$

If i is also $=0$, we have $\theta_0 = M$.

2) $i=0^\circ$. The orbital plane coincides with the tangential plane of the celestial sphere. We have then:

$$(69) \quad \theta_0 = \nu + \omega$$

3) $\omega=0^\circ$ and $\omega=90^\circ$. In these two cases the apparent orbit is symmetrical, and it is sufficient to compute the values for $0^\circ \leq M \leq 180^\circ$. In fact, if θ_0 corresponds to M , then $360^\circ - \theta_0$ will correspond to $360^\circ - M$.

The ephemeris tables are given in the Appendix.

Construction of the curves.

The curves that have been constructed give θ_0 as a function of M for the previously mentioned values of the arguments, with two exceptions: the curves for $e=0.70$ and those for $i=75^\circ$ have not been constructed, since they are of less practical interest.

The curves have been drawn on millimetre paper in the scale of $1^\circ = 1$ mm in both co-ordinates. — To facilitate comparisons with position angle diagrams, the curves have been drawn for a range of 540° in the abscissa.

For practical reasons it is almost necessary to draw several curves on the same paper. The question is then, whether curves for different values of e , i or ω ought to be drawn on the same paper. The three possibilities are illustrated by Figures 15, 16 and 17. In the curves of Fig. 15, the values of e are different, while i and ω are constant. The curves intersect each other, and this proves a disadvantage especially when they are compared with the position angle diagram. The same remark applies to Fig. 16, in which i is varied, while e and ω are constant. In Fig. 17, which gives curves for different values of ω , the different curves do not intersect each other. It is easily understood that this applies to all diagrams of this last type. For that reason the papers, with two exceptions, give curves for constant values of e and i and different values of ω . The two exceptions are the curves for the special cases $e=0$ and $i=0$; each of these sets of curves has been drawn on one paper. The total number of diagrams is thus 26.

It may be added that Figs. 15—17 also illustrate the effect which a change in one of the elements e , i and ω causes on an ephemeris curve.¹¹ The study of such effects will facilitate the comparisons with the position angle diagrams.

¹¹ Cf. Section 4 of the paper reviewed on p. 70—71.

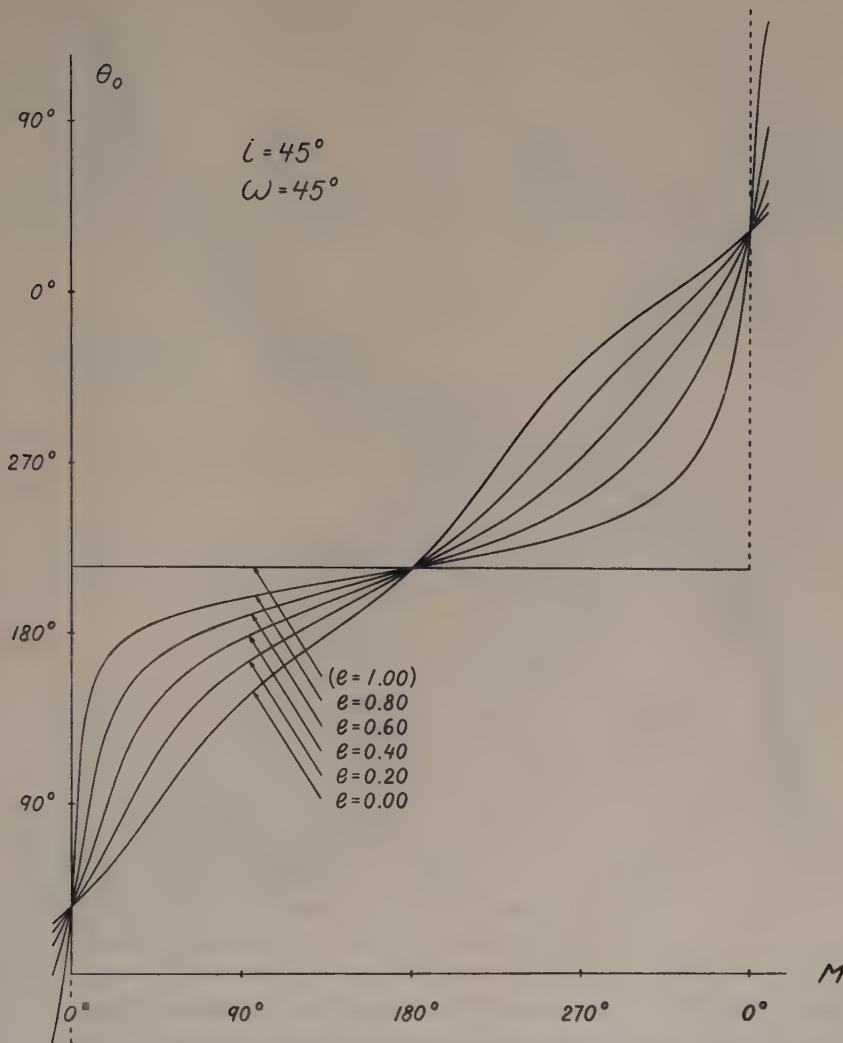


Fig. 15. Ephemeris curves for $i = 45^\circ$, $\omega = 45^\circ$.

There do not seem to be sufficient reasons for printing all the curves in this paper. If they should be printed for practical purposes, the scale ought to be at least $1 \text{ mm} = 1^\circ$.

A few words may be added for the explanation of Fig. 17. The numbers 90° , 75° , . . . to the left, at the bottom of the diagram, give the values of ω , if the origin is $M = 0^\circ$, $\theta_0 = 0^\circ$. The numbers in the parentheses (270° , etc.) indicate that the same curves can also be used if the values of ω are changed by 180° . In this case the M axis is to be transferred 180° upwards; the new

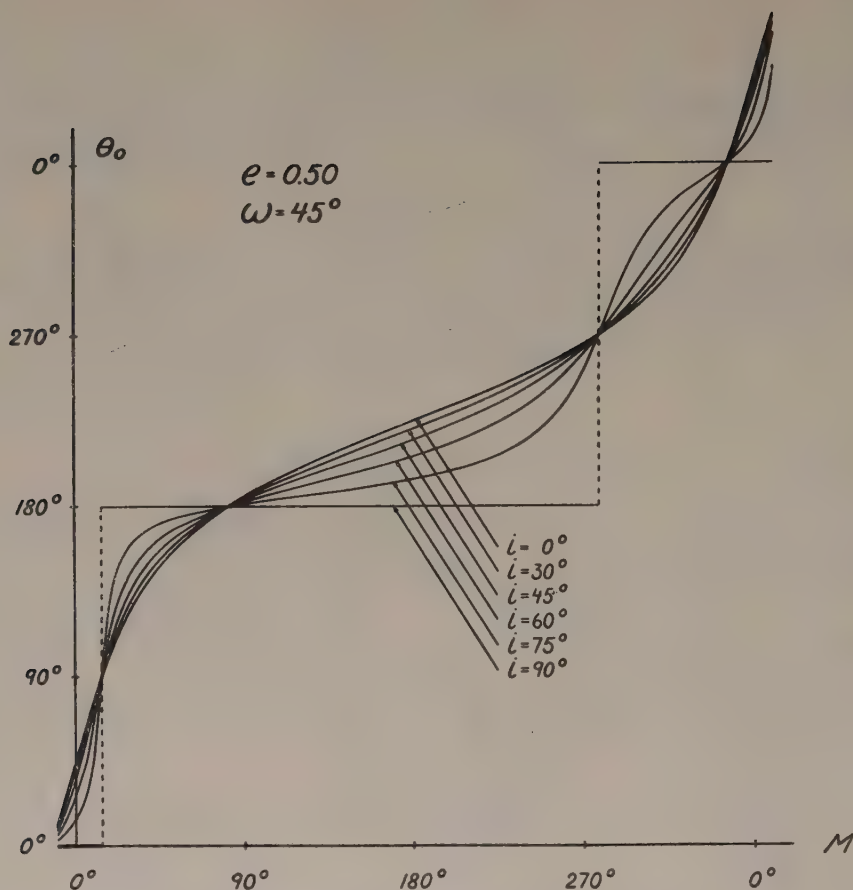


Fig. 16. Ephemeris curves for $e = 0.50$, $\omega = 45^\circ$.

position of the axis is indicated by the lower of the dashed lines. — If the paper is turned upside down and the co-ordinate system is changed accordingly, the values of ω will be 180° , 165° , . . . or 0° , 345° , . . ., in the same way as before.

The orbit determination.

The determination of an orbit according to this method consists of the following stages:

1) An approximate value of the period P is first derived. If this cannot be done from recurrence of angle, an estimate of the period may be obtained by plotting the observations (or selected observations) in polar co-ordinates, drawing the apparent ellipse and making area measurements in the usual

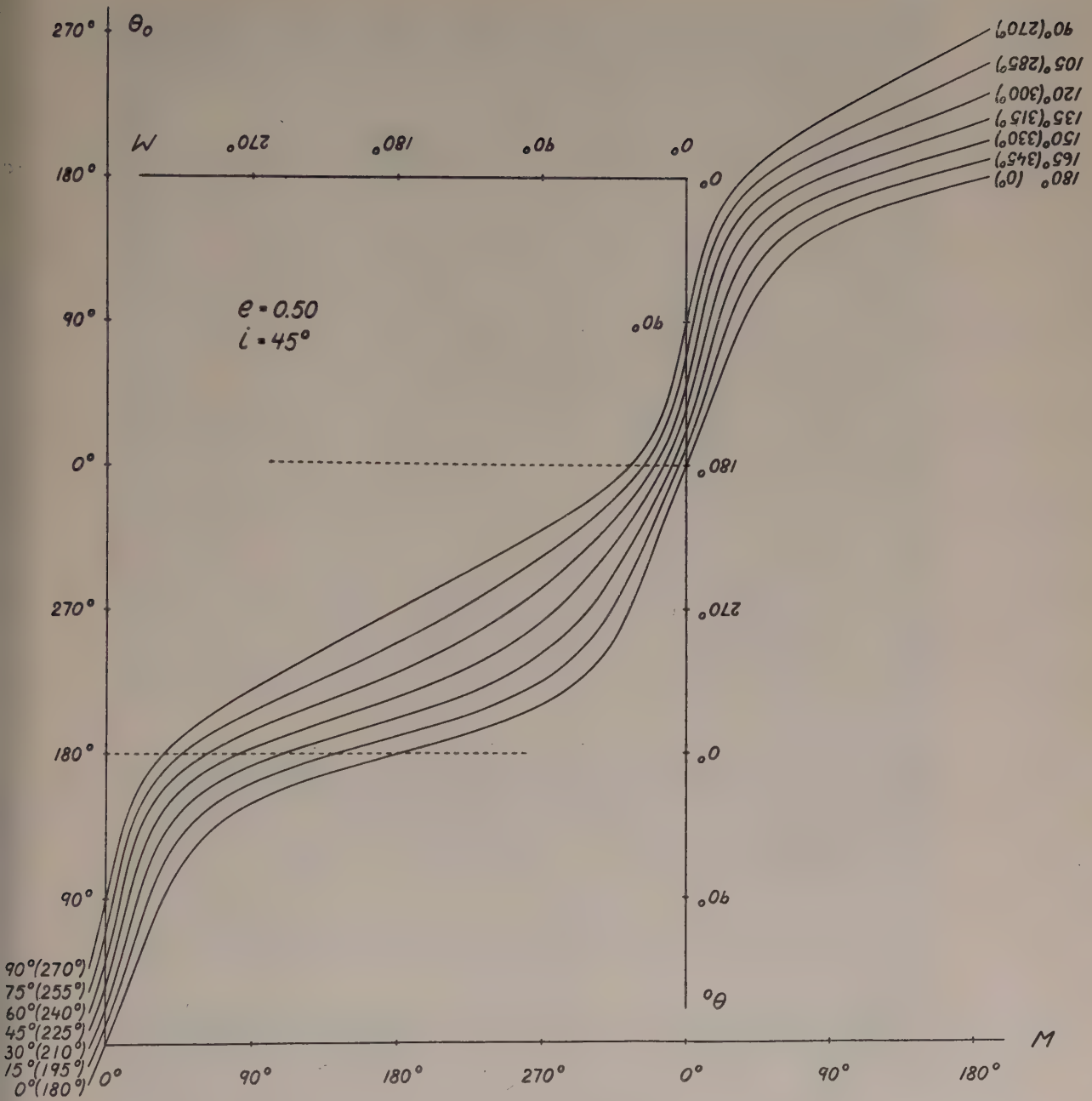


Fig. 17. Ephemeris curves for $e = 0.50$, $i = 45^\circ$.

way. Since the value obtained in this way is often not very accurate, it is advisable to make different assumptions of the value of P .

2) An epoch T' is conveniently chosen, and the values of M' are computed from formula (65) or (66), respectively. Normal places are formed, if the number of observations is comparatively large.

3) The position angle diagram is constructed by plotting θ against M' on millimetre paper in the scale of $1^\circ = 1 \text{ mm}$. The diagram is copied on a transparent paper which is placed upon the millimetre paper; the co-ordinate axes are accurately drawn on the transparent paper. It should be observed that it is not necessary to draw any interpolation curve. On the contrary, it seems to be an advantage not to have drawn it when the diagram is compared with the ephemeris curves. In fact, it may happen that the interpolation curve affects the computer's judgement unfavourably.

4) The transparent paper is placed upon one of the papers with ephemeris curves, and it is examined if any of the curves fits the observations. It is permitted to translate the paper horizontally or vertically, but care must be taken that the axes of the two diagrams are always kept parallel to each other. Then the transparent paper is turned upside down (face still upwards), and the process is repeated. All the papers with ephemeris curves are tried in this way, until the best possible fit is obtained.

It is sometimes found that the (imagined) position angle curve is a little steeper than the ephemeris curve, but that the fit would otherwise have been very good. This indicates that the assumed value of the period is too large, and a better approximation can be obtained by comparing the steepness of the two curves.

5) From the curve giving the best fit the values of e , i and ω are immediately obtained. The co-ordinates of the origin of the position angle diagram, referred to the co-ordinate system of the diagram of ephemeris curves, are read off accurately. From these co-ordinates the values of T and Ω are easily obtained.

For retrograde motion, the value $180^\circ - i$ is substituted for i , and $360^\circ - \omega$, for ω .

If Ω comes out larger than (or equal to) 180° , both Ω and ω are changed by 180° .

6) The value of a can be obtained in the usual way, by computing an ephemeris for the distances, the provisional value $a = 1''$ being used. The scale is then determined by comparing these ephemeris values with reliable distance measurements (or normal places).

An ephemeris of the relative distance values may also be computed in a simple way by using the tables of θ_0 . The accuracy obtained in this way will generally be quite sufficient. The necessary formula is derived in the following way.

The expression of the areal constant in the apparent orbit may be written:

$$(70) \quad c = \rho^2 \frac{d\theta}{dt} = \rho^2 \frac{d\theta}{dM} \frac{2\pi}{P}$$

where θ and M are to be measured in the same unit. On the other hand, we have the following expression of c :

$$(71) \quad c = \frac{2\pi a^2 \cos \varphi \cos i}{P}$$

Combining these expressions, we may write:

$$(72) \quad \rho = \sqrt{10} \cdot a \sqrt{\cos \varphi \cos i} \cdot \frac{1}{\sqrt{10 \frac{d\theta}{dM}}} = K \cdot \rho_1$$

where

$$(73) \quad K = \sqrt{10} \cdot a \sqrt{\cos \varphi \cos i}, \text{ and } \rho_1 = \frac{1}{\sqrt{10 \frac{d\theta}{dM}}}$$

Approximate values of $10 \frac{d\theta}{dM}$ are obtained by interpolation between the differences given in the tables of θ_0 , and thus the values of ρ_1 can be computed. The quantity K can be determined from the measurements of ρ and the computed values of ρ_1 . Then a is determined from K .

In the procedure outlined above, the distance measurements have been used only for the determination of a . However, if these measurements are considered to be reliable enough, they should also be taken into account earlier by using the law of areas. This may be done by the following device which has already been recommended *i.a.* by TH. N. THIELE in his paper on γ Virginis.¹²

The expression of the areal constant may be written:

$$(74) \quad d\theta = c \cdot \frac{1}{\rho^2} dt$$

By integrating from the time t_1 to the time t , corresponding to the position angles θ_1 and θ , we obtain:

$$(75) \quad \theta - \theta_1 = c \int_{t_1}^t \frac{1}{\rho^2} dt$$

¹² Note 2, p. 7.

The function $\frac{1}{\rho^2}$ is represented graphically as a function of t , and the integral in the right member is determined by area measurement for suitable values of t . Thus a relation between θ and t is obtained. The constants in this relation may be determined so that two values (*e.g.* the values at the ends of the measured interval) coincide with those obtained directly from θ and t . The directly measured values of θ can then be compared with those computed from ρ , and necessary adjustments can be made. — Of course, M' may be used as the abscissa equally well as t .

Evidently, the above method of obtaining the elements has the advantage of being rapid and simple. If an ephemeris curve does not give a satisfactory representation, it is generally easy to see in which direction the elements ought to be changed. In the cases of well observed double stars, it is even often possible to interpolate between different ephemeris curves.

Another advantage has already been touched upon: the observations can be directly compared with the theoretically correct curves, no empirical adjustments being necessary. In this connection it will be of interest to recall the following statement by H. N. RUSSELL:¹³

»Curves drawn to represent the observations may play an important part in the solution. An experienced computer is likely to draw these curves nearly of a theoretically possible shape, but, even so, the object of the solution is to fit the observations rather than an empirical curve.»

To this it may be added that the plot of the observations often shows small irregularities, and it is sometimes difficult to decide whether such irregularities correspond to the real course of the orbit, or if they are due to observation errors, or if perhaps a third disturbing component may be present. There is little doubt that such deviations are generally caused by errors of observation, but in some cases the inflection points of the theoretical curve are situated near to each other, and then the curve may have a run which can hardly be foreseen by the computer. In such cases the above method will evidently be favourable.

Another advantage is that the method is apt to give a fair idea of the accuracy of the orbit elements. Other orbit methods may often lead to a serious overestimation of the accuracy.

The greatest drawback of the method is evidently that it only affords a discontinuous scale of values of e , i and ω (if the possibility of interpolating between different curves is not taken into account). The method will thus — at least in its present form — only lead to a rough set of elements. Sometimes, the accuracy of the observations will not justify any attempts at improving the orbit, but generally this should be done by

¹³ MN 93, p. 599 (1933).

differential corrections. In general there will hardly be any risk that the preliminary orbit should be unsufficient as the basis of a differential improvement.

It would naturally be desirable to increase the accuracy of the method by using smaller intervals, especially of e and i . The use of a larger scale when drawing the curves (*e.g.* $1^\circ = 2 \text{ mm}$) might also be considered. Both these improvements would, however, imply a rather large additional work.

A disadvantage, which the method, however, shares with almost all orbit methods, is that it cannot suitably be applied to all kinds of orbits. It is thus advisable to abandon the method if there is reason to suspect that either the eccentricity or the inclination exceeds the limits that have earlier been given.

The tables of θ_0 , which have been used for the construction of the ephemeris curves, can also be used with advantage when separate values of θ_0 are desired. In this case linear interpolation will often be insufficient. On the other hand, it will generally be sufficient to take the second difference into account, using Bessel's interpolation formula.¹⁴ For values near the periastron, even this way may prove to be unsatisfactory; then it will be preferable to compute the values according to formula (67).

The ephemeris curve method will be used in the next chapter for computing the orbits of ADS 3098, 6762, 7334, 8419, 8635 and 9397.

Inflection points.

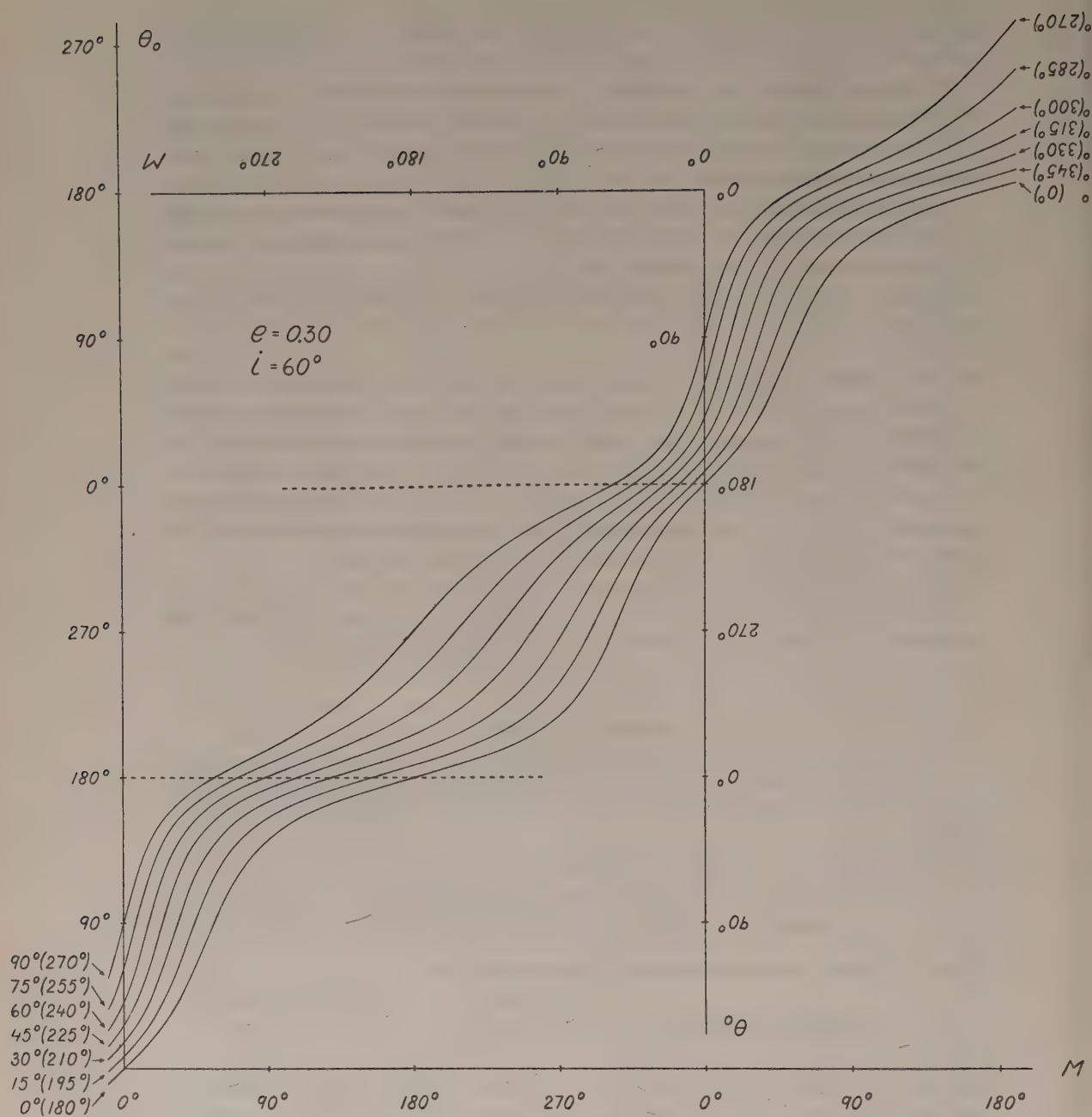
It seems reasonable that it might be useful to study the inflection points of the ephemeris curves, and thus a few reflections will be made on this subject.

From the law of areas it is obvious that an inflection point on the curve of position angles will correspond to a maximum or a minimum of the apparent distance. A simple way of deriving the inflection points is to put $\frac{d\rho^2}{dM} = 0$. This leads to the following expression:

$$(76) \quad \sin^2 i \sin(v + \omega) [\cos(v + \omega) + e \cos \omega] - e \sin v = 0$$

where v is the only variable quantity. By introducing $u = \operatorname{tg} \frac{v}{2}$ we obtain an equation of the fourth degree. In each revolution, a real root of u will give one inflection point. Except in the simple case when both e and i are zero,

¹⁴ See *e.g.* C. WIRTZ, *Tafeln und Formeln*. Berlin (1918). Pages 53 and 193.

Fig. 18. Ephemeris curves for $e = 0.30$, $i = 60^\circ$.

there will be either four or two inflection points in each revolution.¹⁵ (An example of the case of four inflection points is given in Fig. 18.)

For greater convenience, the maximum and the minimum values of the differences in the tables given in the Appendix have been printed in heavy types.¹⁶ This will give a rough idea of the positions of the inflection points.

In practical orbit computation it will not, however, be advisable to lay too much stress on determinations of the inflection points of the position angle curves, or the maximum or minimum points of the distance curves. In fact, such points can seldom be determined with accuracy. It will generally be preferable to study the run of the whole observed arc.

¹⁵ If $i = 0$, there are obviously two inflection points (periastron and apastron). If $i \neq 0$, the number is either four or two. — C. PARVULESCO (Lyon Bull **12**, p. 122 (1930)) erroneously states that the number of inflection points is always four in the latter case.

¹⁶ When two consecutive values are equal, the intermediate value of M has instead been printed in heavy types.

CHAPTER V

The orbits of ten visual binaries

Introduction.

The present chapter contains the orbit determinations of the following ten binaries:

ADS 2612 (AB)	=	BDS 1747	=	Σ 400
» 3098	=	» 2088	=	Σ 511
» 5234	=	» 3474	=	O Σ 149
» 6762	=	» 4570	=	Σ 1216
» 7334	=	JDS 1910	=	A 1342
» 7871	=	BDS 5515	=	O Σ 224
» 8419 (AB)	=	» 6028	=	Σ 3123
» 8635	=	JDS 2205	=	A 1851
» 9397	=		=	A 2983
» 12961	=	JDS 3035	=	A 1658

The intention is to deal especially with binaries whose orbits have not been computed before or have earlier been unsatisfactorily determined. This explains why none of the orbits given here can claim any high degree of accuracy; some of them are of an altogether preliminary character. It is well known, however, that even rather rough orbit determinations may be useful for several purposes.

The measurements have been collected by aid of catalogues of measurements, especially ADS and BDS.¹ Many references to measurements of later years have been taken from the *Astronomischer Jahresbericht*. Whenever possible, the original sources have been consulted. The measurements performed by the different observers have always been given separately, except in very few cases when they have had to be taken from ADS since the original measurements were unpublished.

¹ See references on pp. 20 and 29.

If not otherwise stated, uncorrected values of the measurements have been given, except for Otto Struve. For this observer, the values are corrected according to Pulk Obs 9 (1878) and 10 (1893); the distance values as corrected according to the suggestion in the present paper² are given within parentheses. — The other results obtained in Chapter II have also been taken into account during the computations.

One of the orbits has been computed according to Zwiers' method, one according to Thiele-Innes' method, and six by using the ephemeris curves.³ In one case a parabolic orbit has been obtained by a geometrical method, and in one case a circular orbit has been computed.

Most of the symbols have already been explained in Chapter III.⁴ Other symbols will be explained in the text. The arrangement of the tables of observations and residuals will be explained in the section on ADS 2612.⁵

The designations of the names of the observers are explained in the following list.

List of observers.

A	R. G. Aitken	Dys	F. W. Dyson
B	W. H. van den Bos	Δ	E. Dembowski
Bar	E. E. Barnard	En	R. Engelmann
Big	G. Bigourdan	Fatou	P. Fatou
Bonnet	R. Bonnet	Fox	Ph. Fox
Bow	W. M. Bowyer	Fur	H. Furner
Bru	F. Brünnow	Gled	J. Gledhill
Bry	W. W. Bryant	G Σ	G. Struve
Btz	E. Bernewitz	h	J. Herschel
Bz	P. Baize	Highton	H. P. Highton
β	S. W. Burnham	Hl	A. Hall
Cel	G. Celoria	Howe	H. A. Howe
Cg	W. A. Cogshall	Hu	W. J. Hussey
Chan	E. Chandon	H Σ	H. Struve
Cohn	F. Cohn	Ino	N. Ichinohe
Com	G. C. Comstock	Kui	G. P. Kuiper
Din	W. W. Dinwiddie	L	T. Lewis
Dob	W. Doberck	Lau	H. E. Lau
Doo	E. Doolittle	Lbz	P. Labitzke
Duruy	M. Duruy	Ma	J. H. Mädler

² See p. 42.

³ See Chapter IV.

⁴ Cf. p. 67—68.

⁵ See p. 112.

Maw	W. H. Maw	Schmeidler	F. Schmeidler
Neuj	G. Neujmin	Se	A. Secchi
Nvl	V. Nechvile	Searle	G. M. Searle
Ol	C. P. Olivier	See	T. J. J. See
OΣ	O. Struve	Sp	G. V. Schiaparelli
Per	J. Perrotin	Stone	O. Stone
Pos	A. Postelmann	Σ	W. Struve
Prz	E. Przybyllok	T	K. J. Tarrant
Rabe	W. Rabe	VBs	G. van Biesbroeck
Renz	F. Renz	Wilson, J. M.	J. M. Wilson
RW	R. E. Wilson	Winlock	J. Winlock
Sbk	G. M. Seabroke	Vou	J. Voûte
SBn	S. J. Brown	Wz	C. Wirtz

The orbit of ADS 2612 (AB) = BDS 1747 = Σ 400.

The system of ADS 2612 consists of a close pair AB (magnitudes 6^m.84 and 7^m.8; spectral class F4)⁶ and a distant companion C. The orbit of AB has not been determined earlier. The observed arc extends over more than 190°. Since Wilhelm Struve discovered this pair in 1827, the distance has decreased until about 1930. The periastron has now been passed. The period is evidently very long.

The observations are given in Table 19.

The columns of the table contain: the time of observation; the position angle; the distance; the number of nights; the aperture; the designation of the observer; the error (observed *minus* computed) in position angle; the error in distance.

The expressions (0.53c), *etc.*, in the last column signify computed values.

The position angles given in the table were reduced to the equinox of 1900 by applying the precession correction: $-0^{\circ}.0087(t-1900)$. — The correction for proper motion was neglected since it was found to be insignificant. This also applies to the other binaries in this chapter.

The position angles and the distances were then plotted separately against the time, and interpolation curves were drawn. The position angles and the distances were also plotted in polar co-ordinates. The curves were improved by means of the law of areas, and twenty normal places (each

⁶ The magnitudes are Harvard photometric and have been obtained from Tables 50 and 53 in the paper by H. N. RUSSELL and CH. E. MOORE: *The Masses of the Stars*. Chicago (1940). The spectral class has been taken from the same source. — Unless otherwise stated, the magnitudes and the spectral classes have been obtained in the same way for the other binaries in this chapter.

Table 19. *ADS 2612. Observations and residuals.*

1829.94	282.6	1.53	3n	9 1/2-in.	Σ	+	2.2	+	0.05
30.89	276.5	1.87	2	7	h	—	4.1	+	.40
44.71	288.3	1.50	2	15	O Σ	+	4.0	+	.24
		(1.34)						(+)	.08
45.45	288.1	1.08	2	9 1/2	Ma	+	3.5	—	.17
57.96	286.2	1.06	3	9 1/2	Se	—	3.1	+	.00
58.45	289.8	1.18	2	15	O Σ	+	0.3	+	.13
		(1.05)						(	.00)
59.93	292.2	1.02	3	9 1/2	Ma	+	2.0	—	.01
69.30	295.3	1.13	5,3	7	Δ	—	0.0	+	.25
73.96	295.0	1.2	1	9	Gled	—	3.4	+	.39
79.59	301.9	0.70	2	26	Hl	—	1.2	—	.03
80.10	303.6	1.48	3,2	8	Sbk	—	0.0	+	.76
89.81	306.9	0.65	1	30	H Σ	—	8.0	+	.07
89.93	316.0	0.62	3	26	Hl	+	0.9	+	.04
94.06	317.9	—	1	12	Big	—	3.9	(0.53c)	
98.10	323.0	0.56	3	12	A	—	6.7	+	.08
1902.15	324.6	(0.7)	1	13	Pos	—	14.5	+	.26
04.75	328.9	0.73	2	12	Cg	—	16.8	+	.31
05.63	341.3	0.44	2	36	A	—	6.9	+	.03
08.92	354.2	0.35	3	15	VBs	—	3.8	—	.05
12.42	2.6	0.40	2	36	A	—	6.5	+	.01
13.13	4.9	0.32	2	15	VBs	—	6.5	—	.07
19.67	34.8	0.38	5	40	VBs	+	2.6	—	.02
22.78	41.1	0.31	{2 1}	18 1/2 25 1/2	Fox G Σ	—	0.5	—	.09
24.83	48.7	0.42	4	40	VBs	+	0.9	+	.01
24.94	47.8	0.40	2	18 1/2	Fox	—	0.4	—	.01
25.82	51.7	0.43	1	36	A	+	1.0	+	.02
27.84	57.0	0.42	4	40	VBs	+	0.5	+	.01
29.78	63.8	0.44	2	36	A	+	1.8	+	.03
30.83	69.8	0.40	4	40	VBs	+	4.6	—	.00
30.88	79.1	0.37e	2	10 1/2	Kui	+	13.7	—	.03
37.12	82.8	0.45e	3	10 1/2	Rabe	—	3.0	+	.09
38.13	87.8	0.38e	7	10 1/2	Rabe	—	1.6	+	.02
38.19	99.9	0.29 $\pm$	3	12	Bz	+	10.2	—	.07 $\pm$
39.09	not seen		1	11	Duruy	(93.2c)		(0.35c)	
39.10	99 $\pm$	< 0.3	1	11	Duruy	+	5.7 $\pm$	(0.35c)	

Notes. John Herschel's distance measurement of 1830.89 is very uncertain, as being the mean of 2".24 and 1".50. — Otto Struve's position angle of 1844.71 is the mean of two measurements made in 1841 and 1848. O Σ suspects the first of these to be 10° in error; in that case the mean would be 283°.3 and its residual — 1°.0. — The observation 1894.06: the position angle has been changed by 180°. — The values for 1922.78 have been taken from ADS, since G. Struve's measurements are unpublished.

representing a five year interval) were formed. By aid of these normal places the observed arc was drawn as accurately as possible.

Three diameters of the curve were constructed in the usual way, by dividing three sets of parallel chords into equal parts. The diameters were found to be nearly parallel to each other, and thus there seemed to be reason for computing a parabolic orbit.

Since the arc observed after periastron is rather short, the following device was applied. The arc was copied on transparent paper. This paper

was then placed face downwards on the orbit diagram, and its position was adjusted so that the curve drawn on the transparent paper partly coincided with the curve on the orbit diagram, partly formed its continuation. By copying this continuation on the orbit diagram, the drawn part of the parabola was considerably extended.

The line drawn through the primary (S) parallel to the diameters will evidently be the projection of the axis of the parabola, and its intersection (P') with the orbit will be the projection of the periastron point. — By way of trial a chord LSL_1 was inserted so that $LS=SL_1$. Then LL_1 is the projection of the parameter. L comes after P' in the direction of motion. The mid-point of LS is called Q' . Then it can be shown that the co-ordinates of P' (with S as origin) are the Thiele-Innes constants A and B , respectively, and that the co-ordinates of Q' are F and G .⁷

By measurement the following values were obtained:

$$A = +0''.168 \quad B = +0''.366 \quad F = -0''.203 \quad G = +0''.051$$

From the Thiele-Innes constants the values of q (the periastron distance in the orbit plane), i , Ω and ω were computed:

$$q = 0''.405 \quad i = 59^\circ.7 \quad \Omega = 61^\circ.6 \quad \omega = 7^\circ.3$$

For the sake of control, the method given by R. v. D. R. WOOLLEY⁸ was also used. The obtained values of q , i , Ω and ω were found to agree closely with those given above.

The double areal constant was obtained by measuring sector areas; this led to the following value:

$$c = 0.00844 \text{ } \square''/\text{year}$$

The areal velocity in the true orbit is:

$$\sigma = \frac{c \sec i}{2} = 0.00836 \text{ } \square''/\text{year}$$

The »mean motion»:

$$n = \frac{\sigma}{q^2} = 0.0510/\text{year}$$

The time of periastron passage was obtained from area measurements:

$$T = 1931.0$$

The run of the residuals (see Table 19) is not quite satisfactory, but the systematics in the residuals can hardly be eliminated by any changes in the

⁷ Cf. W. S. FINSEN's paper, UOC 95, p. 225 (1936).

⁸ MN 98, p. 380 (1938); reviewed on p. 94—95 in this paper.

elements. The material was regarded as insufficient for the derivation of differential corrections, and the orbit must be considered as preliminary.

The results can thus be summarized as follows:

$$\begin{array}{ll}
 \sigma = 0.00836 \text{ } \square''/\text{year} & A = +0''.168 \\
 n = 0.0510/\text{year} & B = +0''.366 \\
 T = 1931.0 & F = -0''.203 \\
 q = 0''.405 & G = +0''.051 \\
 i = 59^\circ.7 & \\
 \Omega = 61^\circ.6 & \\
 \omega = 7^\circ.3 &
 \end{array}$$

The dynamical parallax of the binary has been computed from the figures given above, following the precepts given by H. N. RUSSELL and CH. E. MOORE in *The Masses of the Stars*.⁹ (This source has also been used for calculating the dynamical parallaxes of the other binaries in this chapter.) The dynamical parallax was found to be $0''.013$, while the Mount Wilson spectroscopic parallax is $0''.018$. The dynamical parallax corresponds to the total mass $3.76 \odot$.

From the above elements an ephemeris of the position angles and the distances was computed. (See Table 20.)

Table 20. *ADS 2612. Ephemeris (equinox of date).*

	$\circ$	"		$\circ$	"
1940.0	96.8	0.34	1952.0	149.4	0.36
42.0	105.3	0.33	54.0	156.7	0.38
44.0	114.3	0.33	56.0	163.1	0.40
46.0	123.5	0.32	58.0	168.8	0.43
48.0	132.7	0.33	60.0	173.6	0.46
50.0	141.4	0.34			

The orbit of ADS 3098 = BDS 2088 = Σ 511.

The orbit of this binary has not been computed earlier. The magnitudes are $7^m.44$ and $7^m.9$ and the spectral class A0. Since Wilhelm Struve's discovery of the pair in 1827 the companion has travelled over an arc of about 145° . The earliest observations, especially in distance, are rather uncertain.

The observations are given in Table 21.

The position angles were corrected for precession by applying the correction: $-0^\circ.0095(t-1900)$.

A preliminary plot showed that the orbit is fairly open, and therefore the ephemeris curve method was used.¹⁰ From area measurements the

⁹ *Loc. cit.*, p. 130—141.

¹⁰ For a description of the method see Chapter IV.

Table 21. *ADS 3098. Observations and residuals.*

1829.52	320.0	0.54	4n	9 1/2-in.	Σ	— 5.4	— 0.10
43.30	320.7	0.46	3	9 1/2	Ma	+ 4.0	— .17
45.14	316.2	0.65	3	15	O Σ	+ 0.7	+ .02
		(0.62)					(— .01)
58.02	302.0	0.38	2	9 1/2	Se	— 4.2	— .20
63.61	294.1	wedgeshaped	6	7	Δ	— 7.6	(0.55c)
65.93	293.7	wedgeshaped	3	7	Δ	— 6.0	(0.54c)
66.28	283.5	0.42	2	8 1/2	En	— 15.8	— .12
70.25	294.3	0.60	1	15	O Σ	— 1.1	+ .08
		(0.44)					(— .08)
72.90	287.6	wedgeshaped	2	7	Δ	— 5.1	(0.50c)
77.26	288.6	0.49	1	15	O Σ	+ 0.6	+ .01
		(0.50)					(+ .02)
78.84	288.4	0.49	1	18 1/2	β	+ 2.3	+ .02
79.85	286.8	0.40	3	26	Hl	+ 1.9	— .06
80.11	285.9	0.45e	3,2	8	Sbk	+ 1.4	— .01
81.03	287.0	—	1	12	Big	+ 3.6	(0.46c)
83.07	284.5	0.46	7	7 1/2	En	+ 3.8	+ .02
87.91	273.2	0.43	3	10	T	— 0.7	+ .01
88.93	276.0	0.40e	1	19	Sp	+ 3.6	— .02
90.94	273.3	0.39	3	26	Hl	+ 4.1	— .02
94.13	267.3	0.4 e	1	8	Highton	+ 3.4	+ .01
94.15	271.5	0.5 e	2	8	Sbk	+ 7.6	+ .11
95.68	259.2	[0.4]	2	15	Renz	— 2.1	+ .01
96.08	260.3	0.47e	3	19	Sp	— 0.3	+ .08
1904.83	250.2	0.36	2	40	β	+ 6.4	.00
05.86	250.4	0.41	1	36	A	+ 8.6	+ .05
08.01	241.2	0.38	2	15	VBs	+ 3.8	+ .02
13.10	229.2	0.36	2	15	VBs	+ 2.3	.00
20.92	215.8	0.44	4	40	VBs	+ 4.1	+ .07
21.18	211.2	0.32	2	25 1/2	Btz	0.0	— .05
24.79	206.2	0.40	1	40	VBs	+ 2.1	+ .03
24.99	203.0	0.36e	4	10 1/2	B	— 0.7	— .01
26.52	199.8	0.41	2	40	VBs	— 0.6	+ .03
31.82	192.4	0.41	1	40	VBs	+ 1.7	+ .03
35.73	171.4	0.33 $\pm$	1	12	Bz	— 12.3	— .05
38.13	175.1	0.38e	7	10 1/2	Rabe	— 4.3	.00

Note. The position angle measured by Renz in 1895 has been changed by 180°.

period was estimated to be a little shorter than 200 years, and the preliminary value

$$P=192^y$$

was adopted.

The position angle measurements were combined into mean values, and these were corrected by taking the distances into account in the way described in Chapter IV. The values of M' were obtained from the formula:

$$M' = -1^{\circ}.875(t - 1950)$$

The first three columns in Table 22 give the values for the nine normal places.

Table 22. *ADS 3098. Normal places.*

t	M'	θ	Weight	$\Delta\theta$ 1st appr.	$\Delta\theta$ 2nd appr.
1830.00	225°	326.5°	1	+2.8°	+0.7°
43.33	200	318.0	1	+2.3	+0.8
56.67	175	306.0	1,0	-0.5	-1.7
70.00	150	294.5	1	-0.4	-1.5
83.33	125	280.5	2	+1.0	-0.1
96.67	100	260.0	1	+1.5	+0.5
1910.00	75	234.0	2	+1.3	+0.8
23.33	50	206.7	2	+0.2	+0.5
36.67	25	180.5	1	-2.2	-1.2

The points representing the normal places were copied on transparent paper. Comparison with the ephemeris curves gave the best agreement for the following elements:

$$T=1956.90 \quad e=0.50 \quad i=120^\circ \quad \Omega=164^\circ.0 \quad \omega=60^\circ$$

From these values, together with $P=192^y$ and the provisional value $a_1=1''$ of the semi major axis, an ephemeris was computed for the position angles. The obtained residuals are given in the fifth column of Table 22. Considering the uncertainty of the observations, the representation is rather satisfactory. An attempt was made, however, to improve the orbit by differential correction according to the first of the methods proposed by W. P. HIRST.¹¹ The weights of the position angles, as given in the fourth column of Table 22, were applied. The differential coefficients were computed according to eq. (58), p. 92. The following corrections to the elements were successively obtained:

$$\Delta P=-12^y.8 \quad \Delta i=+0^\circ.2 \quad \Delta \omega=+2^\circ.1 \quad \Delta T=+0^y.15 \quad \Delta e=-0.005 \quad \Delta \Omega=-2^\circ.0$$

The large correction to P indicates that the period is only roughly determined. This was also to be expected, considering the relative shortness of the arc and the inaccuracy of the earlier observations. — The corrections to the other elements are small. It must, however, be remembered that the real accuracy of orbit elements is often not so great as might be expected from the sizes of the corrections. In fact, it is well known that rather different sets of elements will often represent the observations about equally well.

The residuals left by the corrected elements are given in the last column of Table 22. The representation of the elements has evidently been improved. A second differential correction of the elements was not considered to be justified.

¹¹ MN 103, p. 337 (1944); reviewed on p. 90—92 in this paper.

With the value $a_1=1''$ and the corrected values of the other elements, an ephemeris of the distances was computed. The computed values of the provisional distances were compared with the observed values for the normal places. The uncertain distance value for the third normal place was excluded; for the others the same weights as for the position angles were used. The resulting value of a was:

$$a=0''.55$$

With this value the last five normal places are well represented, while the computed values of the earlier distances come out larger than the observed ones. As has already been mentioned, however, the early distance measurements are rather unreliable.

The residuals of the observations from the final orbit are given in Table 21.

The final elements and the Thiele-Innes constants have been collected below.

$$\begin{array}{ll} P=179^y.2 & A=-0''.1692 \\ T=1957.05 & B=+0''.3121 \\ e=0.495 & F=+0''.5023 \\ a=0''.55 & G=-0''.0271 \\ i=120^\circ.2 \\ \Omega=162^\circ.0 \\ \omega=62^\circ.1 \end{array}$$

The dynamical parallax is found to be $0''.011$, and the corresponding mass sum is $3.71 \odot$.

The ephemeris is given in Table 23.

Table 23. *ADS 3098. Ephemeris (equinox of date).*

1940.0	176.0	0.38	1952.5	145.1	0.26
42.5	171.2	0.37	55.0	133.2	0.21
45.0	166.1	0.35	57.5	115.2	0.17
47.5	160.4	0.33	60.0	88.1	0.15
50.0	153.7	0.30			

According to the ephemeris, the companion is now approaching the peristron.

The orbit of ADS 5234=BDS 3474=O Σ 149.

The orbit of this binary (magnitudes $6^m.99$, $9^m.5$; spectral class G2) was computed from a rather short arc by S. DE GLASENAPP¹² in 1889; he

¹² AN 123, col. 185 (1889).

Table 24. *ADS 5234. Observations and residuals.*

	$^{\circ}$	$''$	2n	$9\frac{1}{2}$ -in.	Ma	$+12.1^{\circ}$	$+''$
1843.23	24.9	0.55					
48.23	352.5	0.53	3	15	O Σ	- 6.6	+ 0.12
		(0.49)					+ .10
68.33	316.5	—	3	7	Δ	+ 3.5	(+ .06)
70.15	317.6	—	1	7	Δ	+ 7.6	(0.55c)
74.23	296.2	0.58	1	7	Δ	- 7.9	(0.57c)
77.26	303.7	0.73	2	$8\frac{1}{2}$	Sp	+ 3.5	— .03
82.13	300.7	0.77	3	26	Hl	+ 6.1	+ .09
82.21	296.5	0.83	2	$8\frac{1}{2}$	Sp	+ 2.0	+ .09
84.00	292.0	0.89	5	$7\frac{1}{2}$	En	- 0.6	+ .15
84.18	296.6	0.53	2	15	Per	+ 4.2	+ .19
84.28	305.3	—	2	8	Sbk	+ 13.0	- .17
87.18	293.5	0.66	3	19	Sp	+ 4.1	(0.70c)
87.26	287.1	0.84	1	30	H Σ	- 2.2	— .07
88.08	288.3	0.68	2,1	19	Sp	- 0.3	+ .11
89.13	289.9	0.52	4,1	19	Sp	+ 2.3	— .05
91.14	288.5	0.66	3	26	Hl	+ 2.8	— .22
91.15	287.1	0.66	4,1	19	Sp	+ 1.4	— .09
91.22	277.5	1.12	2,1	6	Maw	- 8.2	+ .09
91.44	285.7	0.70	3	12	Big	+ 0.2	+ .37
93.19	284.3	0.89	2	28	L	+ 0.3	— .05
93.22	285.5	0.72	3,2	19	Sp	+ 1.5	+ .13
94.26	282.1	0.54	1	15	Renz	- 1.0	— .04
94.49	283.9	0.55	4	$15\frac{1}{2}$	Com	+ 1.0	— .23
94.68	277.6	0.78	6,3	30	H Σ	- 5.1	— .22
95.19	286.3	0.6 e	1	15	Renz	+ 4.0	+ .01
96.14	275.8	0.80	2	15	Renz	- 5.7	+ .17
96.24	282.1	0.60	4	$15\frac{1}{2}$	Com	+ 0.7	+ .03
96.26	281.5	0.40	1	8	Sbk	+ 0.1	— .17
97.29	281.7	0.68	2	$15\frac{1}{2}$	Com	+ 1.2	— .37
97.93	277.9	0.81	3	12	Hu	- 2.1	— .10
97.95	279.6	0.58	1	36	Hu	- 0.4	+ .03
98.10	277.7	0.88	3	12	A	- 2.1	+ .20
98.23	282.0	0.63	2	28	L	+ 2.3	+ .10
98.26	280.4	0.57	2	$15\frac{1}{2}$	Com	+ 0.7	— .15
99.11	277.8	0.68	2	28	Bow	- 1.2	— .21
99.11	278.7	0.68	2	28	L	- 0.3	— .10
99.26	276.5	0.61	2	$15\frac{1}{2}$	Com	- 2.4	— .10
99.32	281.0	0.77	3	36	Hu	+ 2.2	— .17
1900.71	277.4	0.60	4	28	L	- 0.3	— .01
01.20	275.9	0.40	3,1	8	Sbk	- 1.3	— .18
01.29	272.9	0.68	2	$15\frac{1}{2}$	Com	- 4.3	— .38
02.01	271.4	0.91	3	18	Doo	- 5.2	— .09
02.07	275.5	0.72	3	$15\frac{1}{2}$	Com	- 1.0	+ .14
02.08	278.7	0.59	2	28	Bry	+ 2.2	— .05
02.30	274.5	0.60e	1	13	H Σ	- 1.8	— .18
03.09	274.5	0.70	2	13	Pos	- 1.1	— .17
03.11	279.3	0.94	1	28	Bow	+ 3.7	— .07
03.15	276.7	0.72	2	19	Wz	+ 1.1	+ .17
03.52	271.3	0.64	3,2	28	L	- 4.0	+ .05
03.67	278.0	0.71	2	15	VBs	+ 2.9	— .13
05.19	267.8	0.80	2	28	Fur	- 6.0	— .06
05.76	269.1	0.90	5,1	$8\frac{1}{2}$	Sbk	- 4.2	+ .04
06.18	270.4	0.64	1	36	A	- 2.6	+ .15
06.23	268.8	0.82	2	$15\frac{1}{2}$	Com	- 4.1	+ .11
06.29	276.0	—	1	12	Ino	+ 3.1	+ .07
07.17	276.0	0.76	3,2	19	Wz	+ 3.9	(0.75c)
08.20	272.6	0.69	4	12	Prz	+ 1.4	+ .02
08.27	265.8	0.60	2	$15\frac{1}{2}$	Com	- 5.3	— .04
10.59	270.8	0.60	8	28	Bow	+ 2.0	+ .13
							— .10

Table 24 — (Continued)

	$^{\circ}$	"	3n	15-in.		$^{\circ}$	"
1912.30	264.3	0.63	3n	15-in.	Neuj	— 2.7	— 0.03
13.14	261.1	0.65	3	10 $\frac{1}{2}$	Vou	— 4.9	+ .01
15.18	259.4	0.51	2	15 $\frac{1}{2}$	Com	— 3.8	— .07
15.92	260.5	0.55	2	40	VBs	— 1.5	.00
16.20	256.8	0.50	2	15 $\frac{1}{2}$	Com	— 4.6	— .04
18.21	255.8	0.50	1	15 $\frac{1}{2}$	Com	— 1.5	+ .06
19.27	susp. elong. 260 $^{\circ}$	—	1	15 $\frac{1}{2}$	Com	+ 6.0	(0.37c)
21.15	256.6	0.62	1	28	Fur	+ 14.5	+ .41
21.16	234.6	0.64	5	25 $\frac{1}{2}$	Btz	— 7.4	+ .43
23.25	75.5	0.60	1	28	Fur	— 49.6	+ .51
24.22	91.0	0.20	2	40	VBs	— 3.2	+ .02
24.87	87.2	0.26	3	40	VBs	+ 0.2	+ .02
25.50	83.9	0.27	2	40	VBs	+ 0.2	— .01
25.88	56.9	—	1	25 $\frac{1}{2}$	GΣ	— 24.8	(0.31c)
27.08	79.7	0.55 ±	1	12	Fatou	+ 4.2	+ .17 ±
27.09	71.1	0.63	1	28	Fur	— 4.3	+ .25
27.98	75.1	0.40	2	40	VBs	+ 2.5	.02
29.11	72.6	0.65	1	12	Bonnet	+ 3.0	+ .20
30.04	62.2	0.44	4,1	10 $\frac{1}{2}$	Kui	— 5.1	— .03
30.18	67.5	0.50e	2	12	Bonnet	+ 0.5	+ .03
31.58	68.6	0.58	2	40	VBs	+ 4.7	+ .09
33.26	56.7	0.58	2	12	Bz	— 3.7	+ .08
35.18	53.3	0.58	3	12	Bz	— 3.3	+ .07
37.23	49.7	0.52	1	12	Bz	— 1.4	+ .01
38.17	46.9	0.61	3	12	Bz	— 3.8	+ .11
38.17	49.8	0.67	6	11	Duruy	— 0.9	+ .17
38.18	49.6	0.58	10,9	10 $\frac{1}{2}$	Rabe	— 1.1	+ .08

Notes. The observation 1877.26: distance estimated at 0".80 (2n). — 1882.21: est. 0".70 (2n). — 1887.18: est. 0".63 (3n). — 1888.08: est. 0".53 (2n). — 1889.13: est. 0".50 (4n). — 1891.15: est. 0".60 (2n). — 1893.22: est. 0".72 (2n). — 1894.68: est. 0".53 (3n). — 1921.16: According to ADS, the position angle 134°.6 given by Btz is probably a clerical error. — The position angles of the following measurements have been changed by 180°: 1923.25, 1925.88, 1927.08, 1927.09, 1929.11, 1930.04 and 1930.18. — 1933.26 and 1935.18: »certainly 1st quadrant». — 1938.18: the distance value has been corrected by Rabe. Uncorr. 0".67; est. 0".58 (8n).

obtained the value of 85 $^{\circ}$.9 for the period. After the periastron passage, which took place about 1923, the pair has been measured *i.a.* by G. van Biesbroeck with the 40-inch refractor. In 1927 VAN BIESBROECK¹³ computed a new orbit with the period 103 $^{\circ}$.

When the residuals from von Biesbroeck's orbit are computed for the recent observations,¹⁴ they are found to be positive and rather large both in angle and in distance, and thus there seems to be reason for recomputing the orbit.

The observations are given in Table 24.

¹³ Yerkes Publ 5:1, p. 205 (1927).

¹⁴ In the ephemeris given by van Biesbroeck the position angles should be diminished by about 1°.

The precession correction $-0^{\circ}.0065(t-1900)$ was applied to the observed position angles.

Owing to the high values of e and i , the ephemeris curve method could not be used here. Nor does Zwiers' method seem appropriate, since a large part of the orbit near periastron has not been sufficiently observed. For that reason Thiele-Innes' method was applied.

The position angles and the distances were plotted against the times. The thirteen normal places given in the first three columns of Table 25 were formed.

Table 25. *ADS 5234. Normal places.*

t	θ	ρ	Weight	$\Delta\theta$ 1st appr.	$\Delta\theta$ 2nd appr.
1846.56	$3^{\circ}.6$	0.51	1	$+ 4.6$	$- 0.4$
72.59	309.2	0.63	2,1	$+ 1.4$	$+ 2.7$
83.16	298.7	0.76	2	$+ 3.4$	$+ 5.2$
89.75	287.4	0.68	4	$- 0.1$	$+ 0.3$
95.07	281.9	0.68	4	$- 0.3$	$- 0.5$
98.66	279.3	0.70	4	$+ 0.6$	$- 0.1$
1902.35	275.6	0.71	4	$+ 0.4$	$- 0.7$
07.15	270.5	0.73	4	$+ 0.2$	$- 1.6$
14.78	260.3	0.57	3	0.0	$- 3.4$
20.76	251.5	(0.59)	2,0	$+ 13.7$	$+ 5.7$
24.86	87.2	0.23	4	$+ 1.1$	$+ 0.4$
29.18	71.0	0.53	4	$+ 5.3$	$+ 1.8$
37.07	50.5	0.60	4	$+ 2.6$	$- 2.2$

The normal places were adjusted so as to satisfy the law of areas, and the following value of the double areal constant was determined:

$$c = -0.00919 \text{ } \square''/\text{year}$$

The orbit determination according to Thiele-Innes' method was based on this constant and the following three adjusted normal places:

1889.75	$287^{\circ}.40$	$0''.722$
1914.78	$260^{\circ}.30$	$0''.550$
1924.86	$86^{\circ}.20$	$0''.230$

The computation led to the following elements:

$P=112^y.7$	$A=-0''.2365$
$T=1923.05$	$B=+0''.2206$
$e=0.734$	$F=+0''.2668$
$a=0''.833$	$G=+0''.7761$
$i=110^{\circ}.5$	
$\Omega=74^{\circ}.7$	
$\omega=280^{\circ}.4$	

With these elements an ephemeris was computed for the thirteen normal places. The residuals in angle, which are given in the fifth column of Table 25, are not satisfactory, and therefore an improvement was made according to Hirst's first method. The weights given in the fourth column were used, and the following corrections were successively obtained:

$$\Delta i = -3^{\circ}.3 \quad \Delta e = +0.012 \quad \Delta T = +0^{\circ}.07 \quad \Delta P = -1^{\circ}.7 \quad \Delta \omega = +0^{\circ}.8 \quad \Delta \Omega = +3^{\circ}.5$$

The new residuals are given in the last column of Table 25. The residuals in angle are now more satisfactory. A new value of a was computed in the same way as for ADS 3098. The result was:

$$a = 0''.889$$

The values obtained in this way were adopted as final elements; they have been collected below, together with the Thiele-Innes constants.

$P = 111^{\circ}.0$	$A = -0''.2175$
$T = 1923.12$	$B = +0''.2222$
$e = 0.746$	$F = +0''.2291$
$a = 0''.889$	$G = +0''.8430$
$i = 107^{\circ}.2$	
$\Omega = 78^{\circ}.2$	
$\omega = 281^{\circ}.2$	

The curves in Fig. 19 have been computed from the above elements. The dots represent the observed position angles and distances for the normal places.

The residuals left by the same elements are given in Table 24. The representation of the position angles is, on the whole, quite satisfactory. On the other hand, several of the residuals in distance are large. For the most part, this seems to be due to the observations, which are very discordant.

Among the measurements of late years, those made by Furner and Bernewitz give large residuals in both co-ordinates. The distance residuals for these observations are positive. This is generally the case especially in Furner's measurements, but here the residuals are exceptionally large. In this case, the distances cannot even be approximately correct, since the areal constant would then come out much too large. These measurements have therefore been rejected.

All recent distance measurements (since 1930.18) give positive residuals, but it seems that this cannot be corrected without disturbing the representation of the earlier measurements too much. It should be observed that nearly all of the measurements since 1930 have been performed with instruments of moderate sizes. Further measurements with large instruments are

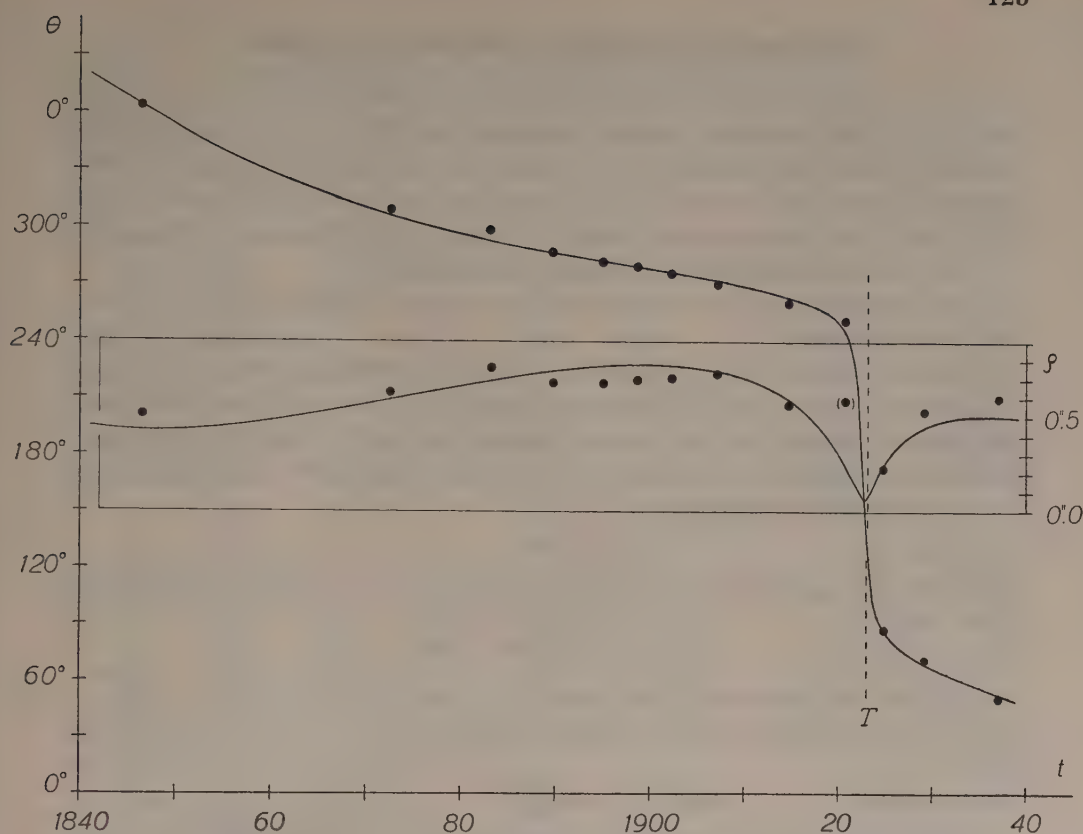


Fig. 19. ADS 5234: position angles (dots outside the rectangle; scale to the left) and distances (dots inside the rectangle; scale to the right) as functions of the time. The dots represent normal places. T = time of periastron passage.

no doubt necessary for a more definite determination of the orbit of this difficult binary.

The dynamical parallax is found to be $0''.030$, which is in good agreement with the value $0''.029$ of the trigonometric parallax given in the *Yale Catalogue*,¹⁵ and also with the Mount Wilson spectroscopic parallax, which is $0''.027$. — The total mass of the system is $2.03 \odot$.

The ephemeris is given in Table 26.

Table 26. ADS 5234. Ephemeris (equinox of date).

	$^{\circ}$	$''$		$^{\circ}$	$''$
1940.0	47.1	0.50	1952.0	19.4	0.44
42.0	43.0	0.49	54.0	14.1	0.43
44.0	38.7	0.48	56.0	8.7	0.43
46.0	34.2	0.47	58.0	3.2	0.43
48.0	29.5	0.46	60.0	357.7	0.43
50.0	24.6	0.45			

¹⁵ F. SCHLESINGER: General Catalogue of Stellar Parallaxes. Yale University Observatory (1935).

The orbit of ADS 6762 = BDS 4570 = Σ 1216.

The magnitudes of this double star are $6^m.81$ and $7^m.5$; its spectral class is A0. The orbit was already computed in 1895 by J. E. GORE,¹⁶ who obtained a period of $174^y.73$. It is now known that the period must be considerably longer, but even now its value is very uncertain. The observed arc is only 120° , and thus the orbit computed below is of a preliminary character.

The observations have been collected in Table 27.

No precession correction was applied; its value would only have been $-0^\circ.0046(t-1900)$.

The ephemeris curve method was used. As has already been mentioned, the period is uncertain. Ten normal places were formed, and for these the position angles were plotted on transparent papers, with the following alternative values of the period: $P=310^y$; 345^y ; 380^y ; 400^y ; 420^y ; 435^y ; and 450^y . It was found that the value

$$P=435^y$$

gave the best agreement. The following formula gives the corresponding values of M' :

$$M' = +0^\circ.82759(t-1800)$$

The normal places and the weights are given in Table 28.

The comparison with the ephemeris curves led to the following elements:

$$T=1842.9 \qquad e=0.20 \qquad i=45^\circ \qquad \Omega=133^\circ.2 \qquad \omega=0^\circ$$

The semi major axis was determined according to the method described on p. 105. The following value was found:

$$a=0''.654$$

The ephemeris computed with the above elements is given in the last column of Table 28. The residuals are small, except for the first two normal places. However, no orbit would give a good representation of the first normal places, and their residuals are by no means impossible.

No considerable improvement could be expected from differential corrections, and therefore the derived elements were kept. They are given below, together with the Thiele-Innes constants.

$$\begin{array}{ll} P=435^y & A=-0''.4479 \\ T=1842.9 & B=+0''.4770 \\ e=0.20 & F=-0''.3373 \\ a=0''.654 & G=-0''.3167 \\ i=45^\circ & \\ \Omega=133^\circ.2 & \\ \omega=0^\circ & \end{array}$$

¹⁶ AN 137, col. 329 (1895).

Table 27. ADS 6762. *Observations and residuals.*

1825.20	109.5	0.53	1n	9 1/2-in.	Σ	— 7.6	+ 0.02
31.24	115.2	0.45	4	9 1/2	Σ	— 7.5	— .07
43.26	178.3	0.8	2,1	9 1/2	Ma	+ 44.8	+ .28
45.19	146.8	0.45	6	9 1/2	Ma	+ 11.5	— .07
51.28	139.5	0.49	2	15	O Σ	— 1.2	— .03
		(0.54)					(+ .02)
53.25	148.3	—	1	9 1/2	Ma	+ 5.8	(0.52c)
55.24	152.1	0.57	2	9 1/2	Ma	+ 7.8	+ .06
57.34	148.0	0.4 $\pm$	1	9 1/2	Se	+ 1.7	— .11 $\pm$
63.80	150.5	—	10	7	Δ	— 1.9	(0.50c)
65.28	151.4	—	1	9 1/2	Se	— 2.4	(0.50c)
66.01	170.3	0.52 $\pm$	7,6	8 1/2	En	+ 15.8	+ .02 $\pm$
67.12	130.6	—	3	15	Winlock	— 25.0	(0.50c)
67.64	133.0	—	2	15	Searle	— 23.1	(0.50c)
73.13	165.3	0.4 e	1	12	Bru	+ 3.7	— .09
73.22	169.4	0.4 e	2,1	8	Wilson, J. M.	+ 7.7	— .09
73.41	153.9	—	4	7	Δ	— 8.0	(0.49c)
74.17	162.0	0.3 e	1	8	Wilson, J. M., and Sbk	— 0.7	— .19
74.18	167.0	0.5 e	1	9	Gled	+ 4.3	+ .01
75.28	165.5	—	2	8	Sbk	+ 1.7	(0.48c)
78.19	167.5	—	2	11	Stone	+ 0.6	(0.48c)
78.22	167.3	0.46	4	8 1/2	Sp	+ 0.4	— .02
78.54	159.5	0.53	6	18 1/2	β	— 7.8	+ .05
78.58	168.1	—	5	11	Howe	+ 0.8	(0.48c)
80.22	166.4	0.62e	1	11	Stone	— 2.7	+ .15
80.32	167.6	—	2	8	Sbk	— 1.6	(0.47c)
80.91	166.7	0.36	3	26	Hl	— 3.1	— .11
86.27	166.7	—	2	8	Sbk	— 9.1	(0.46c)
87.62	174.5	0.46	8,4	19	Sp	— 2.9	.00
88.26	173.2	—	1	8 1/2	Cel	— 4.9	(0.46c)
91.16	180.6	0.42	1	12	Big	— 0.9	— .04
91.21	181.7	0.40	3	10	T	+ 0.1	— .06
91.26	183.2	0.46	4	26	Hl	+ 1.6	+ .01
93.25	186.0	0.35	2,3	13	L	+ 2.0	— .10
94.86	183.9	0.39	8	15 1/2	Com	— 1.8	— .06
95.70	183.0	0.45e	2	15	Renz	— 3.6	.00
96.23	184.5	0.87	1	28	Dys	— 2.7	+ .42
97.95	193.6	0.53	2	12	Hu	+ 5.8	+ .09
97.95	190.9	0.47	1	36	Hu	+ 3.1	+ .03
98.10	191.2	0.64	3	12	A	+ 1.8	+ .20
98.28	193.5	0.45	6	15 1/2	Com	+ 3.9	+ .01
98.38	189.4	0.59	4	18	Doo	— 0.3	+ .15
1900.70	197.2	0.45	4	28	Bry	— 0.2	+ .01
02.39	192.8	0.41	7	15 1/2	Com	— 1.6	— .03
04.23	195.3	0.46	2	15	VBs	— 2.4	+ .02
06.18	198.1	0.36	2	36	A	— 2.1	+ .08
08.20	205.2	0.53	2	26	Ol	+ 2.4	+ .09
08.20	204.6	0.54	2	26	RW	+ 1.8	+ .10
08.27	202.5	0.31	1	28	Bry	— 0.4	— .13
09.13	201.0	—	2	19	Wz	— 3.0	(0.44c)
10.57	206.6	0.36	3	36	A	+ 0.7	— .08
11.18	203.9	0.48e	2	10 1/2	Vou	— 2.7	+ .04
11.22	209.9	0.47	1	19	Wz	+ 3.2	+ .03
12.26	212.0	0.48	2	15 1/2	Com	+ 4.0	+ .04
14.29	214.8	0.36	3	28	Bry	+ 4.1	— .08
14.42	213.5	0.50	4	15	VBs	+ 2.7	+ .06
15.23	213.8	0.44	2	15 1/2	Com	+ 1.9	.00
15.25	211.1	0.44	4	8	Rabe	— 0.8	.00

Table 27 — (Continued)

1916.92	213.9	0.47	1n	18-in.	Ol	— 0.2	+ 0.03
17.08	220.0	0.43	4	18	Doo	+ 5.7	— .01
18.29	219.8	0.40	2	15 1/2	Com	+ 4.0	— .04
18.88	213.2	0.49	4	40	VBs	— 3.4	+ .05
20.92	214.5	0.5 e	4	12	Chan	— 4.7	+ .06
21.66	220.1	0.48	{ 1	25 1/2	Btz	0.0	+ .04
			{ 1	28	Fur		
			{ 1	15 1/2	Nvl		
			{ 1	13	Lbz		
23.23	227.7	0.48	1	36	A	+ 5.6	+ .04
23.27	221.6	—	1	13	Prz	— 0.5	(0.44c)
25.17	228.7	0.50	6,2	10 1/2	B	+ 4.2	+ .05
27.21	224.9	0.49	4	8	Rabe	— 2.1	+ .04
28.24	228.6	0.36	2	28	Fur	+ 0.4	— .09
31.19	234.1	0.47	1	10 1/2	Kui	+ 2.4	+ .01
31.58	231.9	0.40	3	28	Fur	— 0.3	— .06
33.77	224.6	0.63	2	18	Ol	— 10.1	+ .17
38.14	239.5	0.48	4	26 1/2	B	— 0.2	+ .01
38.19	239.1	0.45e	4,2	11	Duruy	— 0.6	— .02
38.27	238.7	0.50e	2	10 1/2	Rabe	— 1.1	+ .03
38.30	232.0	0.58	1	18	Ol	— 7.8	+ .11

Notes. The observations 1843.26, 1867.12 and 1867.64 have been rejected. — 1874.17: »Not separated; only a guess.» — 1878.54: the distance is uncertain; it is the mean of 0".61 (4n) and 0".37 (2n). — 1880.22: corrected by Stone. Uncorr. 165°.8, 0".6e. — 1886.27: »Very doubtful.» — 1896.23: distance rejected. — 1915.25: distance corrected by Rabe. Uncorr. 0".36. — 1921.66: taken from ADS. (The measurements by Btz were not accessible.) — 1931.19: est. distance 0".48 (1n).

Table 28. ADS 6762. Normal places.

t	M'	θ	Weight	$\Delta\theta$
1829.23	24.19	113.3	1,0	— 7.5
51.75	42.83	147.6	1,2	+ 6.4
64.83	53.65	154.8	1,0	+ 1.4
77.37	64.03	165.0	4,2	— 1.0
89.19	73.81	177.1	2	— 2.1
96.56	79.91	188.2	4	+ 0.1
1905.59	87.39	198.8	4	— 0.6
14.65	94.88	212.3	6	+ 1.2
25.27	103.67	225.2	4	+ 0.6
38.19	114.36	239.2	2	— 0.5

Table 29. ADS 6762. Ephemeris.

	$\circ$	"
1940.0	241.7	0.48
45.0	246.9	0.49
50.0	251.8	0.51
55.0	256.4	0.53
60.0	260.7	0.54

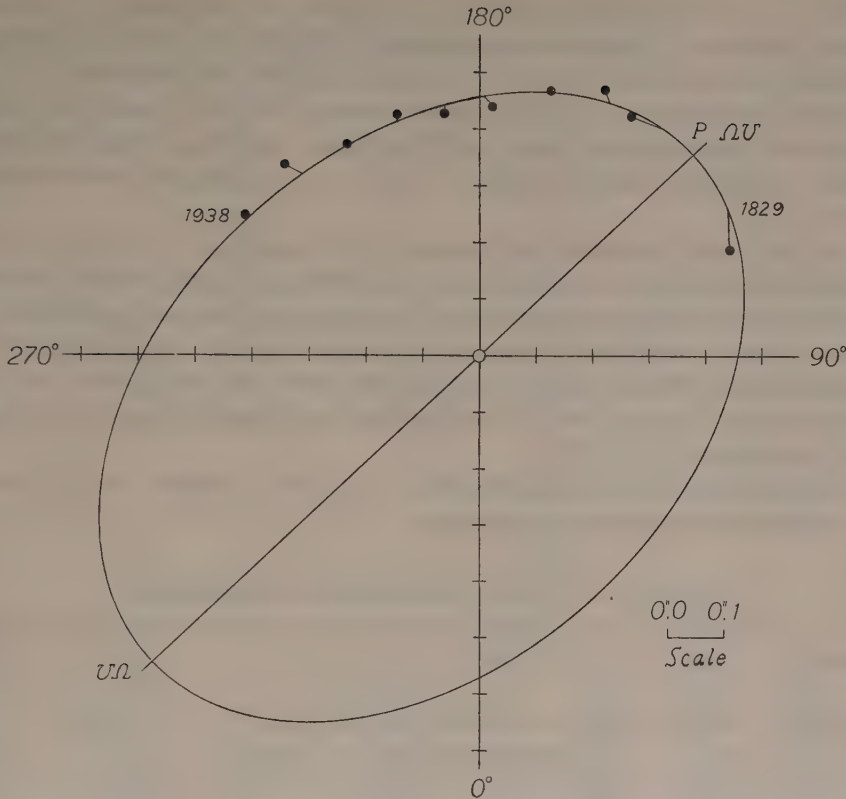


Fig. 20. The apparent orbit of ADS 6762. The dots represent normal places.

The apparent orbit has been drawn in Fig. 20. The dots are observed normal places, and from each dot a line is drawn to the corresponding computed place.

The residuals from the observations are given in the last two columns of Table 27, and the ephemeris in Table 29.

The dynamical parallax is $0''.006$ and the total mass $6.16 \odot$.

The orbit of ADS 7334 = JDS 1910 = A 1342.

This binary (magnitudes according to ADS $7^m.28$, $7^m.3$, spectral class A2) has an unusually short period. In 1934 N. VORONOV¹⁷ published a preliminary orbit with a period of $16^y.50$. This orbit was based on only four annual means, corresponding to the time 1906—23, during which the companion made approximately one revolution. Voronov did not indicate

¹⁷ Tashk Circ 24 (1934).

whether he considered the motion to be direct or retrograde. In FINSEN's *Second Catalogue*¹⁸ the motion is assumed to be direct; in the orbit catalogue compiled by the present author¹⁹ it is assumed to be retrograde. In any case, even the residuals of the four annual means used by Voronov are found to be quite unsatisfactory. After 1923 the binary has been observed during another revolution, and thus there seems to be reason for redetermining the orbit.

To the orbit computer this binary is of more than ordinary interest. The period is now fairly well known. On the other hand, the angular distance is so small that, with the very largest instruments, the pair is difficult to measure even at maximum distance. The measurements, although made by Aitken with the 36-inch and by van den Bos with the 26 $\frac{1}{2}$ -inch, will give large residuals from any orbit. Van den Bos also remarks that the pair was »too close» during 1931–34.

The precession correction, which is $-0^{\circ}.0037(t-1900)$, was neglected. Table 30 contains the observations.

Table 30. *ADS 7334. Observations and residuals.*

	$^{\circ}$	"	3n	36-in.	A		$^{\circ}$	"
1906.22	28.6	0.15	3n	36-in.	A	4	-17.7	0.00
16.31	213.8	0.14	2	36	A	4	+19.2	-.01
18.12	155.8	0.12	2	36	A	4	+2.6	+.02
22.98	4.8	0.14	1	36	A	2	-20.9	-.04
29.28	198:	0.10:	1	36	A	1	-29.7	-.02
31.08	206?	0.15?	1	26 $\frac{1}{2}$	B	1	+7.9	.00
32.15	183?	0.15?	1	26 $\frac{1}{2}$	B	1	+3.4	+.02
33.28	126?	0.13?	1	26 $\frac{1}{2}$	B	1	-21.1	+.04
34.36	41?	0.12?	1	26 $\frac{1}{2}$	B	1,0	-56.1	+.03
36.19	46	0.15	3	26 $\frac{1}{2}$	B	4	-0.8	.00
37.23	35	0.15	4	26 $\frac{1}{2}$	B	4	+1.6	-.02
38.19	37	0.14	4	26 $\frac{1}{2}$	B	4	+13.4	-.04

Since the observations are few, they were used directly; no normal places were formed. The weights assigned are given in the seventh column of Table 30.

The ephemeris curve method was used. The following value of the period was determined from recurrence of angle:

$$P=15^{\gamma}.0$$

From the later observations it is evident that the motion is retrograde. Accordingly, the values of M' were computed from the formula

$$M' = -24^{\circ}(t-1934) \begin{matrix} 1919 \\ 1949 \end{matrix}$$

¹⁸ UOC 100, p. 466 (1938).

¹⁹ Lund Medd Ser II, 94 (1938).

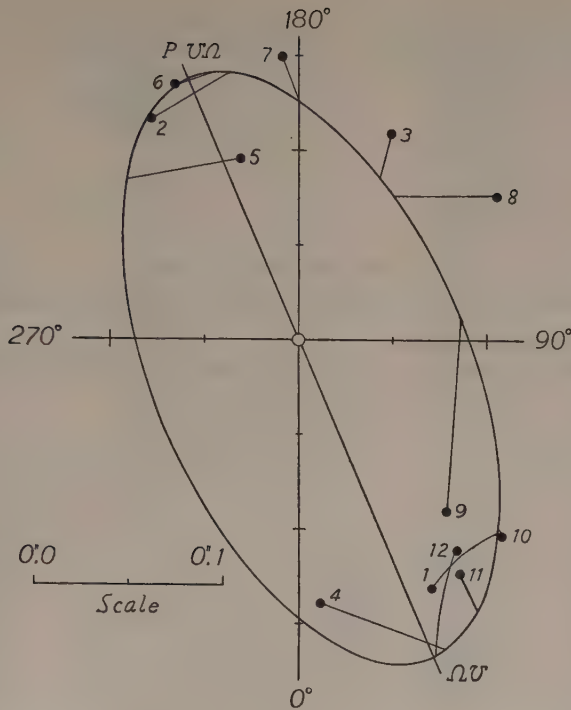


Fig. 21. The apparent orbit of ADS 7334. The dots represent the observations, which are numbered in chronological order.

where the constant in the parenthesis was chosen conveniently in each case.

From comparisons with the ephemeris curves the following elements were obtained:

$$T=1930.75 \quad e=0.10 \quad i=120^\circ \quad \Omega=23^\circ \quad \omega=180^\circ$$

As usual, the value of a was determined by means of a comparison between the observed distances and those computed for $a=1''$. The following value was found:

$$a=0''.166$$

The residuals from the above elements are given in the last two columns of Table 30. As might be expected, they are in general rather large in both co-ordinates. The most serious discordance is found for the single night's observation by van den Bos in 1934. As has already been mentioned, the double star was then too close for reliable measurement.

In any case, the observations are so uncertain that there is hardly anything to be gained by applying differential corrections.

The elements and the Thiele-Innes constants are given below:

$$\begin{aligned}
 P &= 15^y.0 & A &= -0''.1529 \\
 T &= 1930.75 & B &= -0''.0649 \\
 e &= 0.10 & F &= -0''.0325 \\
 a &= 0''.166 & G &= +0''.0765 \\
 i &= 120^\circ \\
 \Omega &= 23^\circ \\
 \omega &= 180^\circ
 \end{aligned}$$

In Fig. 21 the apparent orbit has been drawn.

The ephemeris is given in Table 31. It has been computed for an interval of 10° in mean anomaly.

The dynamical parallax is $0''.019$ and the total mass $3.19 \odot$.

Table 31. *ADS 7334. Ephemeris.*

t	M	θ	ρ	t	M	θ	ρ
		$^\circ$	"			$^\circ$	"
1939.92	220°	4.8	0.16	1945.33	350°	209.2	0.15
40.33	230	358.8	0.14	45.75	0	203.0	0.15
40.75	240	351.6	0.13	46.17	10	196.8	0.15
41.17	250	342.4	0.12	46.58	20	190.2	0.14
41.58	260	330.5	0.10	47.00	30	182.8	0.13
42.00	270	314.9	0.09	47.42	40	173.9	0.12
42.42	280	295.9	0.08	47.83	50	162.7	0.10
42.83	290	275.8	0.08	48.25	60	148.3	0.09
43.25	300	257.7	0.09	48.67	70	130.2	0.08
43.67	310	243.3	0.10	49.08	80	110.1	0.08
44.08	320	232.1	0.12	49.50	90	91.1	0.09
44.50	330	223.2	0.13	49.92	100	75.5	0.10
44.92	340	215.8	0.14				

The orbit of ADS 7871 = BDS 5515 = OΣ 224.

According to RUSSELL and MOORE (*The Masses of the Stars*), the magnitudes of the components of this binary are $8^m.1$ and $10^m.1$.²⁰ The Mount Wilson spectral class is F6. The orbit was already computed in 1893 by S. DE GLASENAPP,²¹ who gave two alternative orbits with the periods $223^y.7$ and $144^y.0$. A third orbit was given in 1894 by J. E. GORE,²² with the period $96^y.13$. The ephemerides of these orbits have been given by W. J. HUSSEY;²³ they show large deviations from the recent observations. On the other hand, the observed arc is now about 120° , and it seems justified to compute a preliminary orbit.

The observations, which are given in Table 32, were not corrected for precession; the correction would have been only $-0^\circ.0021(t-1900)$.

²⁰ The magnitude difference seems to be uncertain; in other sources it is given as $0^m.7$, $1^m.0$ or $1^m.4$.

²¹ *Astronomy and Astro-Physics* **12**, p. 702 (1893).

²² *Astronomy and Astro-Physics* **13**, p. 559 (1894).

²³ *Lick Publ* **5**, p. 102 (1901).

Table 32. ADS 7871. Observations and residuals.

	$^{\circ}$	"	2n	$9\frac{1}{2}$ -in.		$^{\circ}$	"
1843.22	13.7	0.35			Ma	- 7.3	- 0.06
44.31	20	wedgeshaped	1	15	OΣ	+ 1.1	(0.42c)
45.30	13.6	0.20	1	$9\frac{1}{2}$	Ma	- 3.4	- .22
51.27	352.6	0.48	1	15	OΣ	- 13.8	+ .03
		(0.47)					(+ .02)
51.28	17.5	0.25	1	$9\frac{1}{2}$	Ma	+ 11.2	- .20
57.34	3.7	wedgeshaped	1	$9\frac{1}{2}$	Se	+ 7.0	(0.47c)
61.26	348.8	0.59e	1	15	OΣ	- 2.3	+ .10
68.03	339.2	$0.5 \pm$	4,1	7	Δ	- 2.8	- .01±
71.31	328.4	0.59e	1	15	OΣ	- 9.5	+ .07
72.31	336.8	0.55	1	15	OΣ	+ 0.1	+ .02
		(0.46)					(- .07)
73.23	329.8	wedgeshaped	4	7	Δ	- 5.8	(0.53c)
79.32	315.7	0.35e	4,3	$8\frac{1}{2}$	Sp	- 12.7	- .19
80.16	343.3	0.62	1	$18\frac{1}{2}$	β	+ 15.9	+ .08
81.81	312.9	-	3	13	Dob	- 12.6	(0.54c)
83.71	330.2	0.54	7,6	$7\frac{1}{2}$	En	+ 7.0	.00
84.21	326.9	0.55	4	15	Per	+ 4.2	+ .01
87.27	315.6	0.53	4	19	Sp	- 3.6	- .01
88.26	316.9	0.55	2	19	Sp	- 1.1	- .01
92.37	313.6	0.48	4	36	β	+ 0.4	- .05
93.22	314.9	0.55	2	?	Wilson	+ 2.7	+ .02
93.30	308.9	0.50e	1	19	Sp	- 3.2	- .03
93.30	306.0	0.61	2	28	L	- 6.1	+ .08
94.25	312.7	0.43	3	$15\frac{1}{2}$	Com	+ 1.8	- .10
94.75	309.6	$0.52 \pm$	6	30	HΣ	- 0.7	- .01±
95.28	310.7	0.48e	3	19	Sp	+ 1.0	- .05
95.52	309.9	0.41	4	$15\frac{1}{2}$	Com	+ 0.5	- .12
96.29	313.7	0.51	4	28	L	+ 5.2	- .01
97.34	306.8	0.50e	1	19	Sp	- 0.4	- .02
97.35	303.4	0.69	3	12	A	- 3.8	+ .17
98.07	305.8	0.51	5	36	Hu	- 0.5	- .01
98.27	308.6	0.70	1	12	Hu	+ 2.6	+ .18
99.33	304.9	0.54	4	26	SBn	+ 0.2	+ .02
99.34	309.6	0.40	4	28	Bry	+ 4.9	- .12
99.66	302.1	0.60	3	26	See	- 2.2	+ .08
1900.23	298.9	0.35e	1	13	Cohn	- 4.7	- .17
02.29	294.4	0.56	1	26	Din	- 6.5	+ .05
02.30	299.6	0.50	1	13	HΣ	- 1.3	- .01
02.91	301.2	$0.5 e$	3	13	Pos	+ 1.1	- .01
03.27	300.7	0.59	3	28	L	+ 1.1	+ .08
06.04	301.3	0.61	3	19	Wz	+ 7.5	+ .11
07.78	286.0	0.56	2	$15\frac{1}{2}$	Com	- 7.5	+ .07
08.36	281.6	$0.4 \pm$	2,1	12	Prz	- 11.1	- .09±
08.75	293.5	0.44	3	15	VBs	+ 1.3	- .05
12.24	286.4	0.50	2	36	A	- 0.7	+ .02
13.38	283.5	0.41	2	28	Bow	- 2.1	- .06
14.32	292.8	0.66	1	28	Fur	+ 8.8	+ .19
15.03	288.0	0.42	12	8	Rabe	+ 5.1	- .05
15.22	279.6	0.50	1	36	A	- 3.0	+ .04
15.25	284.6	0.46	3	15	VBs	+ 2.0	.00
15.34	277.1	0.48	6	$15\frac{1}{2}$	Com	- 5.3	+ .02
16.22	286.8	0.61	4	18	Doo	+ 5.8	+ .15
18.33	274.4	0.39	7	$15\frac{1}{2}$	Com	- 3.2	- .06
21.04	266.8	0.50	1	36	A	- 7.9	+ .05
22.06	274.5	0.46	1	$15\frac{1}{2}$	Nvl	+ 3.1	+ .02
22.34	267.7	0.43	3	28	Fur	- 3.2	- .01
23.75	271.6	-	2	13	Prz	+ 3.3	(0.44c)
23.82	258.0	0.46e	4	$10\frac{1}{2}$	B	- 10.3	+ .02

Table 32 — (Continued)

1924.20	268.9 ^o	0.47 ["]	3n	40-in.	VBs	+ 1.3 ^o	+ 0.03 ["]
25.34	265.2	—	1	6	Dob	— 0.4	(0.43c)
27.89	263.5	0.55	3	28	Fur	+ 2.2	+ .13
28.30	264.1	0.40	1	12	Fatou	+ 3.5	— .02
30.88	251.5	0.40e	1	10 1/2	Kui	— 3.7	— .02
31.19	254.0	0.42	1	40	VBs	— 0.6	.00
31.30	267.3	0.47	1	28	Fur	+ 12.9	+ .05
33.28	248.2	0.42	4	26 1/2	B	— 2.3	+ .01
34.09	252.5	0.46	5	12	Bz	+ 2.5	+ .05
36.30	242.2	0.44	4	26 1/2	B	+ 2.4	+ .03
39.27	243.4	0.45	2,1	10 1/2	Schmeidler	+ 4.8	+ .04

Notes. The distance value for 1883.71 has been corrected by Engelmann. Uncorr. 0".50. — 1887.27: est. distance 0".47 (3n). — 1888.26: est. distance 0".50 (2n). — 1893.22: the observer is probably H. C. Wilson, Cincinnati. The aperture is then 11". — 1908.36: »unsicher, länglich»; »nur zeitweise getrennt». — 1923.75: »länglich»; »nur zeitweise getrennt».

The orbit determination was made according to Zwiers' method. Nine normal places were formed and are given in Table 33.

Table 33. ADS 7871. Normal places.

t	θ	ρ	Weight	$\Delta\theta$ 1st appr.	$\Delta\theta$ 2nd appr.
1844.28	15.8 ^o	0.30 ["]	0	— 6.7 ^o	— 3.2 ^o
55.30	0.7	0.40	2,1	— 1.6	+ 0.9
71.23	333.6	0.46	0,1	— 5.6	— 1.0
84.39	322.8	0.53	1	+ 0.2	+ 0.3
94.11	311.2	0.50	2	+ 0.6	+ 0.1
1901.30	301.4	0.54	2	+ 0.2	— 0.9
15.17	283.3	0.48	2	+ 2.1	+ 0.6
26.05	264.2	0.46	2	+ 1.1	— 0.1
35.43	246.1	0.44	1	— 0.1	— 0.2

As usual, interpolation curves were drawn, and adjustments were carried out so as to make the measurements satisfy the law of areas. The apparent ellipse was then drawn by means of an ellipsograph; the scale was 1"=400 mm.

We introduce the following designations:

S=the position of the primary star.

C=the centre of the apparent ellipse.

P=the projection of the periastron point (in the plane of the apparent orbit).

a' and b' =the projected semi major and minor axes.

x_1 =the position angle of P.

x_2 =the position angle of the projection of the point for which the true anomaly is 90° .

After the necessary constructions were carried out, the following values could be read off:

$$CS=12.6 \text{ mm} \quad a'=CP=180.0 \text{ mm} \quad b'=212.5 \text{ mm} \quad x_1=24^\circ.4 \quad x_2=-52^\circ.1$$

This led to the following five elements:

$$e=0.070 \quad a=0''.559 \quad i=138^\circ.3 \quad \Omega=148^\circ.8 \quad \omega=117^\circ.0$$

with the corresponding values of the Thiele-Innes constants:

$$A=+0''.4098 \quad B=+0''.1859 \quad F=+0''.3272 \quad G=-0''.4202$$

The values of P and T were then derived by a least squares solution according to the method suggested by W. H. VAN DEN BOS.²⁴ The following values were obtained:

$$P=2327.0 \quad T=1843.36$$

The residuals obtained for the normal places with the above elements are given in the fifth column of Table 33. The large residuals for the first and third normal places are probably due mainly to the uncertainty of the early observations, but in any case there seems to be room for an improvement of the elements. This was carried out according to Hirst's first method with the weights given in the fourth column. The first and third normal places were thus not used, but instead the second normal place was given a somewhat higher weight. In the adjustment, the value $\Delta\theta=-2^\circ.5$ was used instead of $-1^\circ.6$. (A further increase of the numerical value of this residual would have been desirable, but this would have impaired the representation of the distances to a high degree.)

It was found that the residual curve showed a clear resemblance to the coefficient curves for e and i , but not to any of the other coefficient curves. Therefore e and i were corrected, while the other elements were left unchanged. Unfortunately, the coefficient curves for e and i are very similar to each other, which makes it impossible to determine these elements accurately. If only e is corrected, the correction is $\Delta e=-0.058$; on the other hand, if only i is corrected, the correction will be $\Delta i=-4^\circ.6$. The following combination of corrections was adopted:

$$\Delta e=-0.029 \quad \Delta i=-2^\circ.3$$

An adjustment of Ω was also made:

$$\Delta\Omega=-2^\circ.7$$

²⁴ UOC 86, p. 266, (b) (1932).

Finally the value of a was redetermined from the distances:

$$a = 0''.549$$

With the new values of the elements, the ephemeris was recomputed; the residuals from the normal places are given in the last column of Table 33. As will be seen, the representation has been considerably improved. The representation of the distances proves to be less satisfactory, but not unacceptable.

The final elements and the Thiele-Innes constants are given below:

$$\begin{array}{ll} P = 232^y.0 & A = +0''.4036 \\ T = 1843.36 & B = +0''.1529 \\ e = 0.041 & F = +0''.3060 \\ a = 0''.549 & G = -0''.4220 \\ i = 136^\circ.0 \\ \Omega = 146^\circ.1 \\ \omega = 117^\circ.0 \end{array}$$

The residuals from the observations are given in Table 32, and the ephemeris in Table 34.

Table 34. *ADS 7871. Ephemeris.*

	$^{\circ}$	$''$
1940.0	237.1	0.41
44.0	229.1	0.41
48.0	221.3	0.42
52.0	213.7	0.43
56.0	206.5	0.44
60.0	199.7	0.45

The dynamical parallax $0''.010$ is obtained; the spectroscopic parallax is also $0''.010$ (Mt. Wilson). The mass sum is $2.91 \odot$.

The orbit of ADS 8419 (AB) = BDS 6028 = Σ 3123.

ADS 8419 consists of the close pair AB (magnitudes $7^m.89$, $7^m.9$; spectral class F5) and the faint companion C. The orbit of the close pair was computed in 1908 by T. J. J. SEE,²⁵ who obtained the value $103^y.3$ for the period. A comparison with later measurements²⁶ shows considerable deviations in both co-ordinates, and thus it seems justified to compute a new orbit.

²⁵ MN **68**, p. 567 (1908).

²⁶ The position angle residuals for 1914.34 and 1923.76, respectively, should be $+0^\circ.8$ and $+10^\circ.9$, instead of $+7^\circ.7$ and $+17^\circ.0$, which is given in ADS. — The ephemeris values for 1923.93 should be $138^\circ.2$, $0''.30$, instead of $143^\circ.4$, $0''.46$ given in Yerkes Publ 5:1, p. 219 (1927).

Since Wilhelm Struve discovered the pair in 1831 the companion has almost completed a full revolution. The early observations are rather uncertain in both co-ordinates, and Wilhelm Struve stated that this binary was the most difficult one of all those given in the *Mensurae Micrometricae*. Later on, there is an interval of some thirty years during which the binary could not be measured. — The observations are given in Table 35.

Table 35. *ADS 8419. Observations and residuals.*

1832.20	109.7	0.3 e	4,3n	9 1/2-in.	Σ	— 7.1	— 0.02
36.73	115.6	0.2 e	2	9 1/2	Σ	+ 7.8	— .09
36.73	119.5	—	2	9 1/2	O Σ	+ 11.7	(0.29c)
40.42	79	obl.	1	15	O Σ	— 20.2	(0.27c)
40.45	91	obl.	1	15	O Σ	— 8.1	(0.27c)
41.41	88.7	0.44e	1	15	O Σ	— 7.8	+ .18
41.56	95.3	0.3 $\pm$	1	9 1/2	Ma	— 0.7	+ .05 $\pm$
42.78	111.3	0.2 $\pm$	1	9 1/2	Ma	+ 18.8	— .04 $\pm$
51.44	elong. in 51°?	—	1	15	O Σ	— 0.1	(0.15c)
58—68		single	4	15	O Σ	—	—
62.95		single	1	7 1/2	Δ	(294.6c)	(0.16c)
81.32		round	1	12	Big	(217.8c)	(0.27c)
81.38	221.9	0.32	2	15 1/2	β	+ 4.2	+ .05
90.53	200.7	0.35e	2	15	H Σ	+ 2.9	+ .04
92.14	200.0	0.37	3	36	β	+ 5.2	+ .05
92.40	191.2	0.25e	1	19	Sp	— 3.2	— .07
95.15	192.2	0.30	6	36	Bar	+ 2.6	— .03
98.96	181.3	0.36	4	36	A	— 2.0	+ .02
1903.44	179.3	0.35	4,3	15	VBs	+ 2.7	— .01
05.26	175.4	0.30	3	36	A	+ 1.5	— .06
05.50	184.4	0.39	2	10	Lau	+ 10.8	+ .03
05.59	188.1	0.39	4,2	12	VBs	+ 14.6	+ .03
07.84	169.5	0.32	10	18	Doo	— 0.8	— .05
09.70	166.9	0.36	8	15	VBs	— 0.9	— .01
11.43	164.8	0.32	2	36	A	— 0.7	— .05
12.38	163.3	0.33	3	15	Neuj	— 0.9	— .04
14.30	171.2	<0.45	1	7 1/2	Rabe	— 9.5	(0.38c)
14.34	159.8	0.34	2	15	VBs	— 1.9	— .04
16.31	157.1	0.34	2	36	A	— 2.0	— .04
21.33	149.7	0.36	2	36	A	— 3.1	— .02
21.69	151.2	—	4	13	Prz	— 1.1	(0.38c)
23.42	150.9	0.37e	4	10 1/2	B	+ 0.7	— .01
23.93	148.8	0.36	5	40	VBs	— 0.7	— .02
30.93	141.1	0.35e	4	10 1/2	Kui	+ 0.5	— .03
34.11	136.7	0.41	3	12	Bz	+ 0.3	+ .04
37.32	134.4	0.4	3	11	Duruy	+ 2.4	+ .04
37.35	132.2	0.44	3	12	Bz	+ 0.3	+ .08
37.40	128.4	0.36e	6	10 1/2	Rabe	— 3.5	.00
38.25	133.6	0.37 $\pm$	3	11	Duruy	+ 2.9	+ .01 $\pm$

Notes. The position angles of the following observations have been changed by 180°: 1832.20, 1836.73 (2 observations), 1840.45, 1841.56, 1842.78, 1851.44 and 1895.15. — 1892.14: »There is hardly any doubt about the quadrant.» — 1914.30: the distance value has been corrected by Rabe. Uncorr. < 0".42.

The precession correction, $+0^{\circ}.00007(t-1900)$, was neglected.

The ephemeris curve method was used. The nine normal places which were formed are given in Table 36. The position angles of the normal places were plotted on transparent paper; the following different values of the period were used: $P=110^y$; 115^y ; and 120^y . The best agreement was obtained for

$$P=115^y$$

with the corresponding values of M' :

$$M' = -3^{\circ}.1304(t-1940)$$

Table 36. *ADS 8419. Normal places.*

t	M'	θ	Weight	$\Delta\theta$
1834.47	330.35 ^o	113.6 ^o	2,0	+ 1.2 ^o
40.92	310.16	88.0	1,0	— 9.8
51.44	277.23	51?	1,0	— 0.1
81.38	183.50	221.9	2	+ 4.2
94.43	142.65	192.4	5	+ 1.6
1906.91	103.58	172.9	5	+ 1.3
13.75	82.17	161.9	5	— 0.6
22.61	54.44	149.9	5	— 1.3
35.89	12.87	134.4	5,3	+ 0.4

A comparison with the ephemeris curves led to the following elements:

$$T=1973.22 \quad e=0.50 \quad i=150^{\circ} \quad \Omega=103^{\circ} \quad \omega=120^{\circ}$$

An attempt was made to improve these elements by differential correction according to Hirst's first method. However, the improvement in the representation of the angles was not of any account, and thus the above elements were retained, with the exception of Ω , whose value was slightly modified. The correction of Ω was:

$$\Delta\Omega = +0^{\circ}.6$$

The value of a was determined according to the method given on p. 105; the weights given for the distances in the fifth column of Table 36 were applied. The following value was obtained:

$$a=0''.280$$

The final elements and the Thiele-Innes constants are given below:

$$\begin{array}{ll} P=115^y & A=+0''.2370 \\ T=1973.22 & B=-0''.0867 \end{array}$$

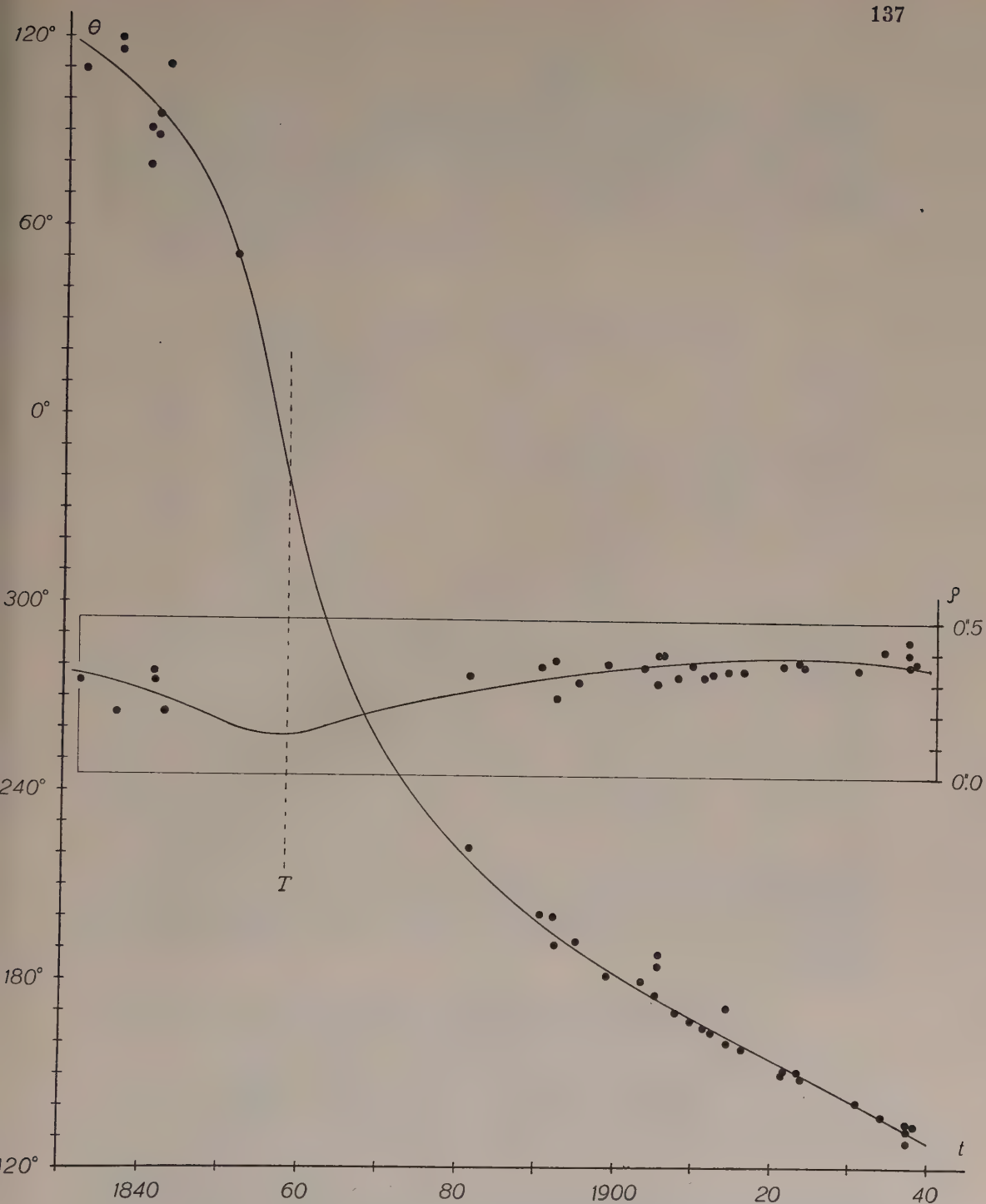


Fig. 22. ADS 8419: position angles (dots outside the rectangle; scale to the left) and distances (dots inside the rectangle; scale to the right) as functions of the time. The dots represent observations. T = time of periastron passage.

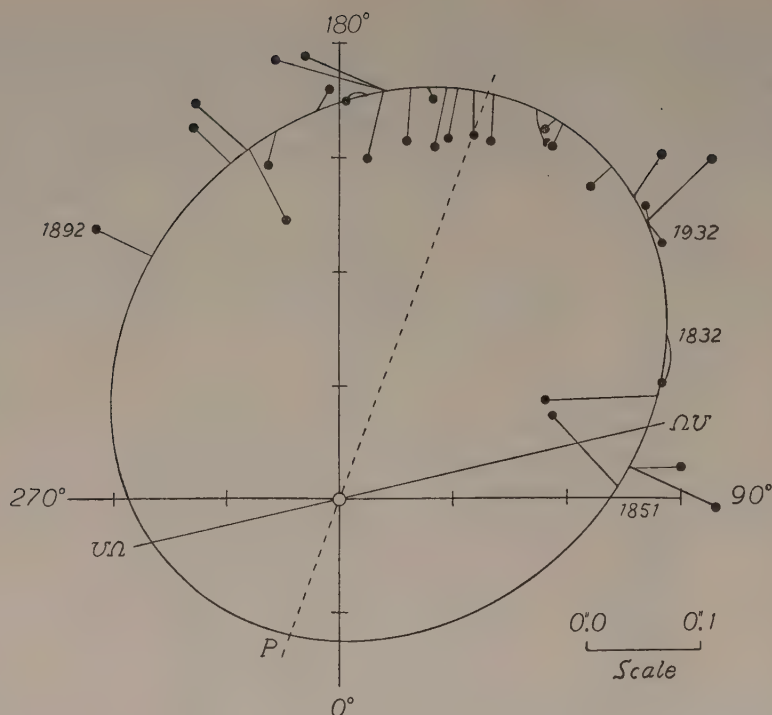


Fig. 23. The apparent orbit of ADS 8419. The dots represent observations.

$$\begin{aligned}
 e &= 0.50 & F &= -0''.0608 \\
 a &= 0''.280 & G &= -0''.2642 \\
 i &= 150^\circ \\
 \Omega &= 103^\circ.6 \\
 \omega &= 120^\circ
 \end{aligned}$$

The residuals of the observations are given in the last two columns of Table 35. The position angles and the distances are given as functions of the time in Fig. 22, and the apparent orbit is shown in Fig. 23. Table 37 gives the ephemeris.

Table 37. ADS 8419. Ephemeris.

1940.0	128.2	0.35	1952.0	107.3	0.29
42.0	125.2	0.35	54.0	102.7	0.28
44.0	122.0	0.34	56.0	97.6	0.26
46.0	118.7	0.33	58.0	91.8	0.24
48.0	115.2	0.32	60.0	85.0	0.22
50.0	111.4	0.30			

The dynamical parallax is $0''.007$, which corresponds to the total mass of $4.64 \odot$. The spectroscopic parallax (Mt. Wilson) is $0''.011$.

The orbit of ADS 8635 = JDS 2205 = A 1851.

The components of this binary have the magnitudes $10^m.2$ and $10^m.6$; the spectrum is K0. No orbit of the binary has been previously published. The observed arc is about 120° , and the apparent orbit is nearly circular. Only nine annual means are available, but nearly all of them have been obtained with large instruments.

Table 38 contains the observations.

Table 38. ADS 8635. Observations and residuals.

1908.38	177.7	0.43	2n	36-in.	A	-0.9	$+0.03$
18.44	229.3	0.42	2	36	A	$+0.1$	.00
21.34	243.8	0.42	2	36	A	$+1.4$	— .01
24.46	253.8	0.42	2	36	A	-2.1	— .02
28.12	272.3	0.47	1	40	VBs	$+1.2$	$+ .02$
31.19	287.1	0.42	1	40	VBs	$+3.8$	— .04
32.14	288.6	0.50	1	40	VBs	$+1.6$	$+ .04$
33.76	285.4	0.56	3	18	Ol	-7.8	$+ .10$
34.36	293.5	0.44	2	40	VBs	-2.0	— .02

The precession correction, $+0''.0010(t-1900)$, was neglected.

Six normal places were formed in the following way. The first four observations were taken as normal places; one normal place was formed by taking the observations 1928.12 and 1931.19 together, (with equal weights), and one obtained from the observations 1932.14 and 1934.36 (weights 1 and 2, respectively). The normal places were given the same weight. — The observation made by Olivier was not included, since it was performed with a smaller instrument than the others, and since it shows deviations in both co-ordinates.

The ephemeris curve method was used, and as usual the position angle measurements were tested by means of the distance measurements in the way described on p. 105—106. Two values of the period were tried, *viz.* $79^y.0$ and $80^y.0$. The latter value:

$$P=80^y.0$$

was adopted; the values of M' were computed from the formula:

$$M' = +4^\circ.5(t-1900)$$

The deviations from the case of circular motion in a plane perpendicular to the line of sight ($e=0$, $i=0$) proved small but quite distinct. The best agreement with an ephemeris curve was obtained for the following elements:

$$T=1902.22 \quad e=0.10 \quad i=0^\circ \quad \Omega=145^\circ.0 \quad \omega=0^\circ$$

As generally is the case when both e and i are small, other sets of elements would suit the observations nearly as well, but, anyhow, the orbit seems to be rather well defined.

The residuals are satisfactory in both co-ordinates; the application of differential corrections would hardly be profitable in this case. The element Ω was, however, diminished by $0^\circ.1$.

The value of the semi major axis was found to be:

$$a=0''.426$$

Thus, the elements and the Thiele-Innes constants are as follows:

$$\begin{array}{ll} P=80^y.0 & A=-0''.3488 \\ T=1902.22 & B=+0''.2451 \\ e=0.10 & F=-0''.2451 \\ a=0''.426 & G=-0''.3488 \\ i=0^\circ & \\ \Omega=144^\circ.9 & \\ \omega=0^\circ & \end{array}$$

The dynamical parallax is $0''.021$ and the corresponding total mass of the system $1.37 \odot$.

The ephemeris is given in Table 39.

Table 39. *ADS 8635. Ephemeris.*

1940.0	316.7	0.47	1952.0	1.6	0.46
42.0	324.1	0.47	54.0	9.4	0.45
44.0	331.5	0.47	56.0	17.4	0.45
46.0	338.9	0.47	58.0	25.5	0.44
48.0	346.9	0.47	60.0	33.9	0.44
50.0	353.9	0.46			

The orbit of ADS 9397 = A 2983.

This binary, for which no orbit has been published before, has the magnitudes $9^m.3$, $9^m.3$ and the spectral class G5. Since the discovery, which was made by Aitken in 1916, the companion has completed a revolution. Therefore it ought to be possible to compute a fairly reliable orbit, in spite of the fact that the observations are rather few, and that the pair is always so close that measurements are difficult even with the largest instruments.

Table 40 contains the observations.

The precession correction, $+0^\circ.0039(t-1900)$, was neglected.

The ephemeris curve method was used. The observations 1933.47 and 1933.56 were combined into one normal place, the weight of the latter

Table 40. *ADS 9397. Observations and residuals.*

1916.40	7.4	0.19	3n	36-in.	A	+ 6.0	+ 0.04
20.47	79.6	0.16	2	36	A	— 6.5	.00
23.45	128.3	0.16	3	36	A	— 0.2	— .03
24.54	143.4	0.20	2	36	A	+ 1.1	+ .01
26.36	167.8	0.16	2	36	A	— 0.4	— .01
27.46	too close		2	36	A	(187.8c)	(0.15c)
33.47	317.1	0.17	1	40	VBs	+ 10.5	— .01
33.56	309.4	0.18	2	36	A	+ 1.4	.00
34.48	319.0	0.15	1	40	VBs	— 2.4	— .03
34.55	319.2	0.20	2	36	A	— 3.2	+ .02
35.43	333.3	0.16	3	36	A	— 2.4	— .01

Table 41. *ADS 9397. Normal places.*

t	M'	θ	Weight	$\Delta\theta$ 1st appr.	$\Delta\theta$ 2nd appr.	$\Delta\theta$ 3rd appr.
1916.40	24.73	7.4	2	— 0.5	+ 8.7	+ 6.0
20.47	96.62	79.6	2	— 6.1	— 4.3	— 6.5
23.45	149.26	128.3	2	+ 0.9	— 1.0	— 0.2
24.54	168.52	143.4	2	+ 2.3	— 0.7	+ 1.1
26.36	200.67	167.8	2	+ 1.9	— 4.0	— 0.4
33.53	327.32	312.0	3	+ 8.0	+ 4.4	+ 4.4
34.48	344.10	319.0	1	— 2.9	— 2.0	— 2.4
34.55	345.34	319.2	2	— 4.0	— 2.8	— 3.2
35.43	360.88	333.3	2	— 6.9	— 1.6	— 2.4

observation being twice that of the former. Each of the other measurements was taken as a normal place. The normal places are given in Table 41. The weights assigned (given in the fourth column) are the same in both co-ordinates.

From three different assumptions about the run of the orbit, the following values of the period were obtained by area measurement: $P=23^y.02$; $21^y.77$; and $20^y.38$. The last of these values,

$$P=20^y.38$$

was found to give the best agreement with the ephemeris curves. The corresponding values of M' :

$$M' = +17^{\circ}.66(t-1915)$$

are given in the second column of Table 41.

The following values were further obtained by comparison with the ephemeris curves:

$$T=1933.85 \quad e=0.10 \quad i=30^{\circ} \quad \Omega=130^{\circ} \quad \omega=180^{\circ}$$

The residuals in position angle left by these elements are given as »1st appr.» in Table 41. Their run is not quite satisfactory, and therefore correc-

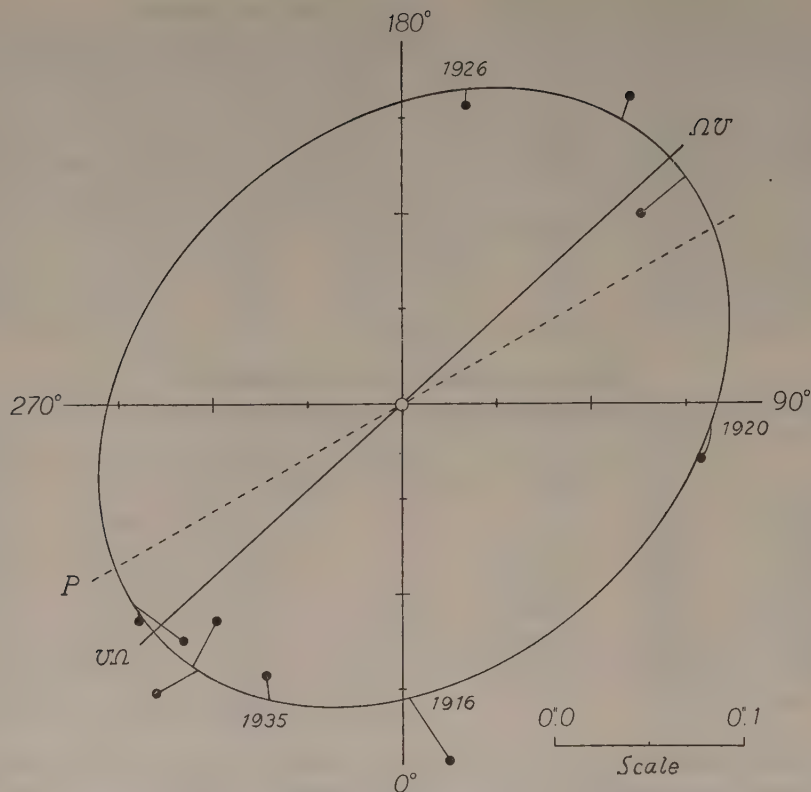


Fig. 24. The apparent orbit of ADS 9397. The dots represent observations.

tions were applied according to Hirst's first method. The following corrections were successively obtained:

$$\Delta\omega = -15^\circ.7 \quad \Delta e = -0.09 \quad \Delta i = +8^\circ.7 \quad \Delta P = +0^y.07 \quad \Delta T = -0^y.87 \quad \Delta\Omega = +2^\circ.1$$

The corresponding values of the elements are:

$$P = 20^y.45 \quad T = 1932.98 \quad e = 0.01 \quad i = 38^\circ.7 \quad \Omega = 132^\circ.1 \quad \omega = 164^\circ.3$$

Some of the above corrections are rather large, and the residuals (given in Table 41 as »2nd appr.») are not very satisfactory. A second improvement was therefore undertaken; it led to the following differential corrections:

$$\Delta e = +0.028 \quad \Delta\omega = -0^\circ.8 \quad \Delta i = +1^\circ.1 \quad \Delta T = 0^y.00 \quad \Delta P = +0^y.05 \quad \Delta\Omega = +0^\circ.1$$

The residuals which result from the new elements are given in the table as »3rd appr.». The agreement is somewhat better than before. With a view to the uncertainty in the observations of this very close pair, there

do not seem to be sufficient reasons for further attempts at improving the orbit.

In the same way as for the other orbits, the distance measurements (which had, of course, also been taken into account earlier) gave the value of the semi major axis:

$$a=0''.184$$

The final elements and the Thiele-Innes constants are as follows:

$$\begin{array}{ll} P=20^{\text{y}}.50 & A=+0''.0890 \\ T=1932.98 & B=-0''.1580 \\ e=0.038 & F=+0''.1358 \\ a=0''.184 & G=+0''.0524 \\ i=39^{\circ}.8 \\ \Omega=132^{\circ}.2 \\ \omega=163^{\circ}.5 \end{array}$$

The last two columns of Table 40 contain the residuals of the observations, and the apparent orbit has been drawn in Fig. 24. The ephemeris (for a total revolution, with mean anomaly intervals of 10°) is given in Table 42.

Table 42. *ADS 9397. Ephemeris.*

t	M	θ	ρ	t	M	θ	ρ
1935.83	50°	342.2°	0.17	1946.08	230°	156.3°	0.18
36.40	60	352.0	0.16	46.65	240	164.8	0.17
36.97	70	2.7	0.15	47.22	250	174.1	0.16
37.54	80	14.2	0.15	47.79	260	184.4	0.16
38.10	90	26.5	0.14	48.35	270	195.8	0.15
38.67	100	39.3	0.14	48.92	280	208.2	0.14
39.24	110	52.0	0.14	49.49	290	221.3	0.14
39.81	120	64.1	0.15	50.06	300	234.7	0.14
40.38	130	75.5	0.16	50.63	310	247.6	0.14
40.95	140	85.8	0.16	51.20	320	259.8	0.15
41.52	150	95.2	0.17	51.77	330	271.0	0.16
42.09	160	103.9	0.18	52.34	340	281.2	0.16
42.66	170	111.8	0.18	52.91	350	290.6	0.17
43.23	180	119.4	0.19	53.48	0	299.4	0.17
43.80	190	126.6	0.19	54.05	10	307.8	0.18
44.37	200	133.8	0.19	54.62	20	316.1	0.18
44.94	210	141.0	0.19	55.19	30	324.4	0.18
45.51	220	148.5	0.18	55.76	40	333.1	0.17

The dynamical parallax is $0''.020$, corresponding to the total mass $1.81 \odot$.

The orbit of ADS 12961 = JDS 3035 = A 1658.

The orbit of this binary (magnitudes $8^m.25$, $8^m.6$, spectral class F5) has not been published before. The arc observed since the discovery in 1907 is about 120° . This ought to be sufficient for the derivation of preliminary elements, since the orbit is nearly circular.

The measurements are given in Table 43. They were used directly; no normal places were formed.

Table 43. *ADS 12961. Observations and residuals.*

1907.51	161.0	0.23	4n	36-in.	A	2	+ 2.0	+ 0.01
17.56	115.9	0.23	2	36	A	2	- 1.9	+ .01
19.90	107.8	0.22	2	36	A	2	- 0.7	.00
22.20	96.2	0.22	5	36	A	3	- 3.1	.00
27.79	71.9	0.17	1	36	A	1	- 5.0	- .05
30.49	73.7	0.20	3	40	VBs	2	+ 6.6	- .02
31.68	62.4	0.22	2	36	A	2	+ 1.0	.00
33.94	53.2	0.23	3	40	VBs	2	+ 0.9	+ .01
34.24	49.6	0.21	2	36	A	2	- 1.5	- .01
37.68	36.3	0.22	4	26 1/2	B	2	- 1.1	.00

The precession correction, $+0^\circ.0052(t-1900)$, was not taken into account during the computations; see, however, Note 27 below.

Most of the observed distances are nearly equal. Further, when the position angles are plotted against the times, it will be seen that they fall nearly on a straight line. The deviations from this line are so small that they may quite possibly be due to the errors of observation alone. The orbit was therefore assumed to be circular ($e=0$, $i=0^\circ$); this corresponds to a linear formula for the position angles:

$$\theta = k(t-1900) + l$$

The constants k and l were determined according to the method of least squares. The weights given in the seventh column of Table 43 were used. The normal equations:

$$\begin{aligned} 1680.30 &= 520.39 k + 20.00 l \\ 37375.34 &= 15127.00 k + 520.39 l \end{aligned}$$

led to the values:

$$k = -4^\circ.00/\text{year} \quad l = 188^\circ.1$$

Thus, the formula for the position angles will be:

$$\theta = 188^\circ.1 - 4^\circ.00(t-1900)$$

This gives the following value for the period:

$$P = 90^y.0^{27}$$

²⁷ If precession is taken into account, this value will instead be $90^y.1$.

The radius of the orbit was computed as the weighted mean value of the measured distances. This mean is:

$$a=0''.218$$

For the dynamical parallax, the value $0''.007$ was found, corresponding to the total mass $4.18 \odot$.

Table 44 gives the ephemeris.

Table 44. *ADS 12961. Ephemeris (equinox of date).*

1940.0	28.1 ^o	} $\rho = 0''.22$
42.5	18.1	
45.0	8.1	
47.5	358.1	
50.0	348.1	
52.5	338.1	
55.0	328.1	
57.5	318.1	
60.0	308.1	

APPENDIX

Appendix

Tables of $\theta_0 = \theta - \Omega$.

The tables give the values of $\theta_0 = \theta - \Omega$ for the following values of the arguments:

$e = 0.00, 0.10, \dots, 0.70$.

$i = 0^\circ, 15^\circ, \dots, 75^\circ$.

$\omega = 0^\circ, 15^\circ, \dots, 90^\circ$.

$M = 0^\circ, 10^\circ, \dots, 360^\circ$.

The differences between the table values for consecutive values of M are also given. The maximum and the minimum values of the differences (or the corresponding values of θ_0) are printed in heavy types.

The first table contains the values of θ_0 for the special case $e = 0.00$; the second table, the values for the special case $i = 0^\circ$. Each of the other tables gives θ_0 for fixed values of e and i and different values of ω .

All the tables, except in the two special cases, have been computed according to the formula

$$\operatorname{tg} \theta_0 = \operatorname{tg} (v + \omega) \cos i$$

For a more detailed account see Chapter IV.

The designations are explained on p. 67—68.

$$\theta_0 = v + \omega.$$

Special case: $i = 0^\circ$.

$i = 0^\circ$ (For other values of ω : add ω to the value in the table.)										
M	$e = 0.00$	$e = 0.10$	$e = 0.20$	$e = 0.30$	$e = 0.40$	$e = 0.50$	$e = 0.60$	$e = 0.70$	M	
0°	0.00	12.27	0.00	0.00	0.00	0.00	0.00	0.00	0°	
10	12.27	12.16	15.25	19.30	25.00	33.34	45.99	65.31	10	
20	24.43	11.98	30.15	37.66	47.61	60.76	77.58	97.64	20	
30	36.41	11.70	44.42	54.44	66.77	81.41	97.84	115.08	30	
40	48.11	11.39	57.88	69.42	82.60	96.96	111.76	126.34	40	
50	59.50	11.03	70.44	82.65	95.72	109.05	122.09	134.50	50	
60	70.53	10.65	82.10	94.34	106.75	118.82	130.22	140.88	60	
70	81.18	10.28	92.91	104.74	116.20	126.98	136.93	146.12	70	
80	91.46	9.93	102.96	114.07	124.47	133.99	142.66	150.60	80	
90	101.39	9.59	112.34	122.54	131.83	140.18	147.69	154.54	90	
100	110.98	9.29	121.14	130.32	138.49	145.74	152.20	158.08	100	
110	120.27	9.03	129.46	137.55	144.62	150.82	156.32	161.92	110	
120	129.30	8.80	137.37	144.33	150.33	155.54	160.15	164.33	120	
130	138.10	8.61	144.95	150.75	155.71	159.99	163.76	167.17	130	
140	146.71	8.45	152.26	156.91	160.84	164.22	167.19	169.88	140	
150	155.16	8.34	159.37	162.85	165.78	168.29	170.50	172.49	150	
160	163.50	8.27	166.33	168.65	170.59	172.26	173.71	175.03	160	
170	171.77	8.23	173.19	174.35	175.32	176.14	176.87	177.53	170	
180	180.00	8.23	180.00	180.00	180.00	180.00	180.00	180.00	180	
190	188.23	8.27	186.81	185.65	184.68	183.86	183.13	182.47	190	
200	196.50	8.34	193.67	191.35	189.41	187.74	186.29	184.97	200	
210	204.84	8.45	200.63	197.15	194.22	191.71	189.50	187.51	210	
220	213.29	8.61	207.74	203.09	199.16	195.78	192.81	190.12	220	
230	221.90	8.80	215.05	209.25	204.29	200.01	196.24	192.83	230	
240	230.70	9.03	222.63	215.67	209.67	204.46	199.85	195.67	240	
250	239.73	9.29	230.54	222.45	215.38	209.18	203.68	198.68	250	
260	249.02	9.59	238.86	229.68	221.51	214.26	207.80	201.92	260	
270	258.61	9.93	247.66	237.46	228.17	219.82	212.31	205.46	270	
280	268.54	10.28	257.04	245.93	235.53	226.01	217.34	209.40	280	
290	278.82	10.65	267.09	255.26	243.80	233.02	223.07	213.88	290	
300	289.47	11.03	277.90	265.66	253.25	241.18	229.78	219.12	300	
310	300.50	11.39	289.56	277.35	264.28	250.95	237.91	225.50	310	
320	311.89	11.70	302.12	290.58	277.40	263.04	248.24	233.66	320	
330	323.59	11.98	315.58	305.56	293.23	278.59	262.16	244.92	330	
340	335.57	12.16	329.85	322.34	312.39	299.24	282.42	262.36	340	
350	347.73	12.27	344.75	340.70	335.00	326.66	314.01	294.69	350	
360	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	360	

$$\theta_0 = M$$

$e = 0.10 \quad i = 15^\circ$								
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	14.51	29.15	44.01	59.13	74.50	90.00	0°
10	11.86	11.96	12.13	12.35	12.56	12.67	12.69	10
20	23.69	11.99	12.21	12.41	12.54	12.59	12.50	20
30	35.47	11.98	12.18	12.34	12.40	12.33	12.17	30
40	47.12	11.86	12.03	12.11	12.08	11.94	11.74	40
50	58.62	11.68	11.78	11.77	11.65	11.46	11.26	50
60	69.90	11.39	11.41	11.32	11.15	10.96	10.79	60
70	80.87	11.03	10.89	10.82	10.64	10.46	10.33	70
80	91.51	10.61	10.49	10.32	10.14	9.99	9.86	80
90	101.78	10.18	10.03	9.85	9.69	9.61	9.50	90
100	111.65	9.74	9.57	9.41	9.30	9.27	9.31	100
110	121.14	9.33	9.17	9.04	8.98	8.99	9.09	110
120	130.28	8.98	8.83	8.74	8.73	8.79	8.92	120
130	139.09	8.65	8.54	8.50	8.53	8.64	8.78	130
140	147.61	8.40	8.33	8.33	8.40	8.53	8.68	140
150	155.91	8.20	8.16	8.21	8.31	8.45	8.60	150
160	164.03	8.06	8.08	8.15	8.29	8.43	8.56	160
170	172.05	7.99	8.03	8.14	8.33	8.46	8.53	170
180	180.00	7.97	8.06	8.19	8.33	8.46	8.52	180
190	187.95	8.01	8.12	8.26	8.41	8.49	8.52	190
200	195.97	8.10	8.24	8.39	8.50	8.56	8.53	200
210	204.09	8.25	8.39	8.53	8.62	8.63	8.56	210
220	212.39	8.43	8.58	8.70	8.75	8.71	8.60	220
230	220.91	8.69	8.83	8.90	8.90	8.82	8.68	230
240	229.72	8.96	9.07	9.11	9.06	8.94	8.78	240
250	238.86	244.95	260.38	275.90	291.37	306.65	321.67	250
260	248.35	254.22	269.72	285.23	300.59	315.72	330.59	260
270	258.22	263.81	279.33	294.77	309.99	324.96	339.68	270
280	268.49	273.74	289.22	304.53	319.59	334.39	348.99	280
290	279.13	284.00	299.38	314.53	329.42	344.07	358.59	290
300	290.10	294.56	309.79	324.76	339.48	354.03	8.52	300
310	301.38	305.40	320.45	335.23	349.82	4.32	18.85	310
320	312.88	316.49	331.34	345.97	0.48	15.00	29.64	320
330	324.53	327.79	342.47	357.00	11.50	26.10	40.90	330
340	336.31	339.26	353.81	8.30	22.87	37.63	52.64	340
350	348.14	350.89	5.38	8.30	22.87	37.63	52.64	350
360	0.00	14.51	29.15	44.01	59.13	74.50	90.00	360

$e = 0.10 \quad i = 30^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	13.06	26.57	40.89	56.31	72.81	90.00	0°	
10	10.67	21.06	38.21	53.42	69.74	86.85	104.10	10	14.10
20	21.47	35.46	50.45	66.57	83.58	100.86	117.68	20	13.58
30	32.57	47.34	63.24	80.11	97.39	114.36	130.42	30	12.74
40	44.00	59.65	76.34	93.59	110.69	126.98	142.16	40	11.74
50	55.78	72.24	89.42	106.63	123.16	138.61	152.97	50	10.81
60	67.79	84.84	102.11	118.88	134.63	149.27	162.98	60	10.01
70	79.84	97.13	114.10	130.18	145.13	159.08	172.35	70	9.37
80	91.69	108.84	125.24	140.55	154.78	168.22	181.26	80	8.91
90	103.10	119.81	135.50	150.07	163.74	176.87	189.90	90	8.64
100	113.88	129.97	144.94	158.90	172.17	185.18	198.37	100	8.47
110	123.98	139.37	153.68	167.17	180.23	193.30	206.81	110	8.52
120	133.38	148.11	161.88	175.06	188.07	201.36	215.33	120	8.66
130	142.15	156.28	169.66	182.69	195.80	209.45	223.99	130	8.84
140	150.37	164.03	177.15	190.18	203.55	217.66	232.83	140	9.04
150	158.15	171.46	184.47	197.64	211.38	226.07	241.87	150	9.25
160	165.61	178.70	191.75	205.18	219.41	234.72	251.12	160	9.40
170	172.86	185.87	199.08	212.91	227.71	243.64	260.52	170	9.48
180	180.00	193.06	206.57	220.89	236.31	252.81	270.00	180	9.48
190	187.14	200.39	214.30	229.21	245.24	262.19	279.48	190	9.40
200	194.39	207.96	222.38	237.91	254.51	271.73	288.88	200	9.25
210	201.85	215.85	230.88	247.03	264.05	281.33	298.13	210	9.04
220	209.63	224.18	239.84	256.54	273.80	290.89	307.17	220	8.84
230	217.85	233.03	249.32	266.42	283.68	300.36	316.01	230	8.66
240	226.62	242.46	259.29	276.57	293.57	309.68	324.67	240	8.52
250	236.02	252.50	269.69	286.89	303.40	318.84	333.19	250	8.47
260	246.12	263.10	280.39	297.23	313.10	327.84	341.63	260	8.47
270	256.90	274.17	291.25	307.51	322.65	336.75	350.10	270	8.64
280	268.31	285.54	302.13	317.66	332.08	345.65	358.74	280	8.91
290	280.16	297.01	312.89	327.65	341.45	354.64	7.65	290	9.37
300	292.21	308.40	323.48	337.53	350.86	3.87	17.02	300	10.01
310	304.22	319.60	333.90	347.38	0.43	9.63	27.03	310	10.81
320	316.00	330.54	344.19	357.31	10.33	10.21	37.84	320	11.74
330	327.43	341.24	354.44	7.45	20.72	11.83	49.58	330	12.74
340	338.53	351.81	4.83	18.00	31.77	12.76	62.32	340	13.58
350	349.33	2.36	15.48	29.10	43.61	13.57	75.90	350	14.10
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	360	

$e = 0.10 \quad i = 45^\circ$

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	10.73	22.21	35.26	50.77	69.25	90.00	0°
10	8.74	9.30	32.73	47.03	65.67	86.14	107.10	10
20	17.81	10.15	44.68	62.04	82.15	103.22	122.72	20
30	27.54	11.36	58.30	77.94	99.03	119.01	136.21	30
40	38.25	12.81	73.42	94.39	114.82	132.68	147.62	40
50	50.20	14.24	89.29	110.09	128.66	144.26	157.39	50
60	63.44	15.10	104.73	124.04	140.41	154.11	165.96	60
70	77.62	15.02	118.72	135.97	150.36	162.66	173.74	70
80	92.06	13.84	130.87	146.10	158.96	170.34	181.03	80
90	105.90	12.38	141.26	154.83	166.61	177.45	188.11	90
100	118.47	10.70	150.19	162.51	173.60	184.24	195.17	100
110	129.54	9.23	158.01	169.47	180.19	190.93	202.43	110
120	139.18	8.07	165.04	175.96	186.61	197.71	210.06	120
130	147.61	7.21	171.52	182.19	193.01	204.75	218.24	130
140	155.09	6.58	177.67	188.34	199.59	212.22	227.12	140
150	161.88	6.16	183.65	194.55	206.48	220.28	236.79	150
160	168.17	5.93	189.63	201.00	213.86	229.09	247.27	160
170	174.16	5.86	195.77	207.85	221.91	238.74	258.44	170
180	180.00	5.93	202.21	215.26	230.77	249.25	270.00	180
190	185.84	6.15	209.12	223.42	240.54	260.47	281.56	190
200	191.83	6.55	216.69	232.48	251.25	272.12	292.73	200
210	198.12	7.11	225.11	242.56	262.72	283.78	303.21	210
220	204.91	7.89	234.56	253.66	274.65	295.05	312.88	220
230	212.39	8.90	245.19	265.62	286.60	305.66	321.76	230
240	220.82	10.11	256.96	278.03	298.12	315.46	329.94	240
250	230.46	11.45	269.62	290.40	308.93	324.48	337.57	250
260	241.53	12.68	282.65	302.22	318.89	332.83	344.83	260
270	254.10	13.53	295.46	313.23	328.07	340.67	351.89	270
280	267.94	13.71	307.56	323.35	336.60	348.20	358.97	280
290	282.38	13.17	318.69	332.65	344.68	355.62	366.26	290
300	296.56	12.17	328.84	341.34	352.51	363.16	373.93	300
310	309.80	11.06	338.20	349.64	360.35	370.84	381.57	310
320	321.75	10.03	346.98	357.80	367.84	378.12	389.77	320
330	332.46	9.26	355.46	365.10	375.16	385.89	397.41	330
340	342.19	8.80	363.94	372.44	382.83	393.16	405.05	340
350	351.26	8.80	372.74	380.80	390.88	400.88	412.77	350
360	0.00	10.73	22.21	35.26	50.77	69.25	90.00	360

$e = 0.10 \quad i = 60^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°	
10	6.21	14.45	24.44	37.88	57.40	84.51	113.51	10	23.51
20	12.80	22.35	34.96	53.11	78.96	108.38	132.26	20	18.75
30	20.24	32.07	48.87	73.19	102.66	128.10	145.87	30	13.61
40	29.14	44.60	67.16	96.20	123.19	142.52	155.85	40	9.98
50	40.33	60.99	89.00	117.35	138.53	153.03	163.59	50	7.74
60	54.74	81.11	110.39	133.69	149.68	161.05	169.98	60	6.39
70	72.76	102.22	127.77	145.64	158.09	167.55	175.56	70	5.58
80	92.92	120.58	140.74	154.59	164.06	173.13	180.73	80	5.17
90	111.94	134.78	150.43	161.62	170.44	178.19	185.75	90	5.02
100	127.49	145.45	157.94	167.44	175.46	183.00	190.85	100	5.10
110	139.41	153.65	164.06	172.51	180.14	187.77	196.27	110	5.42
120	148.58	160.24	169.30	177.14	184.68	192.72	202.26	120	5.99
130	155.84	165.77	173.99	181.55	189.28	198.05	209.13	130	6.87
140	161.82	170.62	178.35	185.92	194.12	204.02	217.29	140	8.16
150	166.97	175.04	182.59	190.40	199.40	210.93	227.21	150	9.92
160	171.58	179.25	186.85	195.19	205.38	219.21	239.36	160	12.15
170	175.86	183.40	191.29	200.49	212.40	229.36	253.87	170	14.51
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180	16.13
190	184.14	192.11	201.50	213.79	231.38	256.64	286.13	190	14.51
200	188.42	197.04	207.78	222.64	244.35	273.00	300.64	200	12.15
210	193.03	202.64	215.37	233.71	259.76	289.13	312.79	210	9.92
220	198.18	209.29	224.82	247.48	276.56	303.47	322.71	220	8.16
230	204.16	217.49	236.83	263.82	292.85	315.42	330.87	230	6.87
240	211.42	227.92	251.87	281.29	307.08	325.17	337.74	240	5.99
250	220.59	241.36	269.46	297.74	318.80	333.22	343.73	250	5.42
260	232.51	258.17	287.61	311.71	328.33	340.05	349.15	260	5.10
270	248.06	277.19	303.96	323.05	336.22	346.07	354.25	270	5.02
280	267.08	295.72	317.41	332.25	342.99	351.60	359.27	280	5.17
290	287.24	311.44	328.14	339.91	349.04	356.90	4.44	290	5.58
300	305.26	323.93	336.85	346.57	354.69	2.24	10.02	300	6.39
310	319.67	333.83	344.20	352.63	0.25	7.89	16.41	310	7.74
320	330.86	341.94	350.71	358.44	6.01	14.23	24.15	320	9.98
330	339.76	348.91	356.79	4.32	12.32	21.75	34.13	330	13.61
340	347.20	355.25	2.79	10.63	19.68	31.30	47.74	340	18.75
350	353.79	1.37	9.08	17.82	28.81	44.13	66.49	350	23.51
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360	

$e = 0.10 \quad i = 75^\circ$										
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	3.97	8.50	14.51	24.15	44.01	90.00	0°		
10	3.22	7.60	13.24	21.93	38.99	79.56	130.04	10		
20	6.71	12.01	19.90	34.59	69.35	122.69	150.33	20		
30	10.81	17.97	30.66	59.73	113.46	146.57	160.66	30		
40	16.10	27.04	50.87	101.86	141.64	158.35	166.93	40		
50	23.72	43.02	88.07	134.98	155.42	165.24	171.33	50		
60	36.21	73.19	125.69	151.55	163.16	169.92	174.77	60		
70	59.06	112.70	146.26	160.51	168.24	173.48	177.70	70		
80	95.62	138.78	157.07	166.18	171.99	176.43	180.38	80		
90	127.90	152.45	163.63	170.24	175.02	179.06	182.98	90		
100	145.98	160.38	168.15	173.42	177.65	181.55	185.67	100		
110	156.09	165.62	171.59	176.11	180.07	184.04	188.59	110		
120	162.45	169.46	174.41	178.52	182.43	186.67	191.96	120		
130	166.93	172.52	176.88	180.80	184.84	189.58	196.09	130		
140	170.36	175.11	179.15	183.07	187.42	192.99	201.51	140		
150	173.17	177.43	181.34	185.43	190.33	197.23	209.21	150		
160	175.62	179.61	183.56	188.00	193.80	202.90	221.15	160		
170	177.86	181.76	185.90	190.95	198.19	211.09	240.80	170		
180	180.00	183.97	188.50	194.51	204.15	224.01	270.00	180		
190	182.14	186.34	191.52	199.10	212.95	245.36	299.20	190		
200	184.38	189.01	195.26	203.49	227.15	275.78	318.85	200		
210	186.83	192.19	200.18	215.18	250.77	303.83	330.79	210		
220	189.04	196.19	207.22	231.31	282.52	321.94	338.49	220		
230	193.07	201.65	218.37	258.18	309.15	332.97	343.91	230		
240	197.55	209.82	237.68	291.09	325.59	340.19	348.04	240		
250	203.91	223.47	268.96	315.45	335.62	345.36	351.41	250		
260	214.02	247.97	301.52	329.85	342.29	349.36	354.33	260		
270	232.10	283.70	322.45	338.72	347.15	352.68	357.02	270		
280	260.38	312.94	334.55	344.76	351.00	355.63	359.62	280		
290	300.94	329.62	342.17	349.28	354.27	358.39	363.02	290		
300	323.79	339.34	347.52	352.95	357.25	361.16	365.02	300		
310	336.28	345.73	351.67	356.17	360.13	364.11	368.02	310		
320	343.90	350.42	355.16	359.19	363.12	367.48	371.02	320		
330	349.19	354.21	358.33	362.24	365.45	369.40	373.33	330		
340	353.29	357.54	361.45	365.55	369.40	373.33	377.02	340		
350	356.78	360.71	364.73	368.94	373.33	377.02	380.00	350		
360	0.00	3.97	8.50	14.51	24.15	44.01	90.00	360		

		$e = 0.20 \quad i = 15^\circ$						M
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	
0°	0.00	14.51	29.15	44.01	59.13	74.50	90.00	0°
10	14.75	29.39	44.26	59.39	74.75	90.26	105.76	10
20	14.55	44.16	59.29	74.65	90.16	105.66	121.02	20
30	14.13	58.54	73.90	89.40	104.91	120.28	135.41	30
40	43.43	13.55	87.81	103.32	118.71	133.87	148.77	40
50	56.98	12.82	100.80	116.22	131.42	146.36	161.06	50
60	69.80	12.03	112.80	128.06	143.06	157.80	172.37	60
70	81.83	11.18	123.82	138.90	153.71	168.31	182.81	70
80	93.01	10.39	133.95	148.85	163.51	178.03	192.53	80
90	103.40	9.65	143.30	158.04	172.60	187.09	201.65	90
100	113.05	8.98	151.97	166.60	181.10	195.62	210.27	100
110	122.03	8.41	160.10	174.65	189.14	203.72	218.49	110
120	130.44	7.92	167.79	182.29	196.81	211.48	226.38	120
130	138.36	7.52	175.12	189.62	204.20	218.98	234.01	130
140	145.88	7.19	182.18	196.71	211.37	226.27	241.43	140
150	153.07	6.95	189.05	203.63	218.40	233.42	248.71	150
160	160.02	6.76	195.80	210.45	225.34	240.49	255.87	160
170	166.78	6.64	202.48	217.23	232.23	247.50	262.95	170
180	173.42	6.58	209.15	224.01	239.13	254.50	270.00	180
190	180.00	6.58	215.86	230.84	246.08	261.53	277.05	190
200	186.58	6.64	222.68	237.78	253.13	268.62	284.13	200
210	193.22	6.76	229.65	244.88	260.31	275.83	291.29	210
220	199.98	6.95	236.84	252.17	267.66	283.17	298.57	220
230	206.93	7.19	244.28	259.71	275.23	290.70	305.99	230
240	214.12	7.52	252.06	267.55	283.06	298.45	313.62	240
250	221.64	7.92	260.21	275.73	291.20	306.48	321.51	250
260	229.56	8.41	268.82	284.33	299.71	314.85	329.73	260
270	237.97	8.98	277.93	293.37	308.62	323.61	338.35	270
280	246.95	9.65	287.60	302.94	318.03	332.85	347.47	280
290	256.60	10.39	297.90	313.08	327.99	342.66	357.19	290
300	266.99	11.18	308.87	323.85	338.58	353.14	7.63	300
310	278.17	12.03	320.54	335.32	349.91	4.41	11.27	310
320	290.20	12.82	332.93	347.55	2.05	12.16	18.94	320
330	303.02	13.55	346.05	359.85	15.07	13.15	31.23	330
340	316.57	14.13	359.85	14.37	13.93	14.14	44.59	340
350	330.70	14.55	14.27	28.90	29.00	43.86	58.98	350
360	345.25	14.75	14.88	44.01	43.76	58.88	74.24	360
	0.00	14.51	29.15	44.01	59.13	74.50	90.00	

M		$e = 0.20 \quad i = 30^\circ$										M
		$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$				
0°	0.00	13.28	13.06	26.57	40.89	56.31	72.81	90.00	17.47	90°	0°	
10	13.28	26.80	13.74	41.14	56.58	73.09	90.29	107.47	16.38	107.47	10	
20	26.70	13.62	41.04	56.47	72.98	90.17	107.36	123.85	14.68	123.85	20	
30	40.32	13.74	55.69	72.15	89.33	106.54	123.07	138.53	12.94	138.53	30	
40	54.06	13.63	70.42	87.55	104.79	121.42	137.00	151.47	11.43	151.47	40	
50	67.69	13.21	84.74	102.01	118.78	134.54	149.18	162.90	10.25	162.90	50	
60	80.90	13.21	98.18	115.12	131.13	146.01	159.91	173.15	9.37	173.15	60	
70	93.36	12.46	110.46	126.77	141.97	156.11	169.49	182.52	8.74	182.52	70	
80	104.88	11.52	121.51	137.08	151.54	165.13	178.23	191.27	8.32	191.27	80	
90	115.39	10.51	131.38	146.24	160.12	173.36	186.37	199.59	8.03	199.59	90	
100	124.90	9.51	140.23	154.49	167.94	180.99	194.07	207.62	7.86	207.62	100	
110	133.55	8.65	148.26	162.02	175.20	188.21	201.50	215.48	7.77	215.48	110	
120	141.44	7.89	155.61	169.02	182.05	195.16	208.77	223.25	7.74	223.25	120	
130	148.72	7.28	162.46	175.62	188.64	201.95	215.96	230.99	7.74	230.99	130	
140	155.51	6.79	168.92	181.96	195.06	208.66	223.14	238.73	7.77	238.73	140	
150	161.94	6.43	175.12	188.13	201.42	215.40	230.39	246.50	7.81	246.50	150	
160	168.11	6.17	181.15	194.24	207.80	222.21	237.73	254.31	7.84	254.31	160	
170	174.10	5.99	187.10	200.35	214.26	229.17	245.20	262.15	7.85	262.15	170	
180	180.00	5.90	193.06	206.57	220.89	236.31	252.81	270.00	7.85	270.00	180	
190	185.90	5.99	199.11	212.95	227.75	243.68	260.56	277.85	7.85	277.85	190	
200	191.89	6.17	205.34	219.58	234.90	251.31	268.46	285.69	7.84	285.69	200	
210	198.06	6.43	211.83	226.55	242.39	259.21	276.49	293.50	7.81	293.50	210	
220	204.49	6.79	218.67	233.91	250.26	267.39	284.63	301.27	7.77	301.27	220	
230	211.28	7.28	225.96	241.75	258.55	275.83	292.85	309.01	7.74	309.01	230	
240	218.56	7.89	232.80	250.14	267.26	284.51	301.15	316.75	7.77	316.75	240	
250	226.45	8.65	242.29	259.11	276.39	293.40	309.52	324.52	7.77	324.52	250	
260	235.10	9.51	251.52	268.68	285.90	302.47	317.97	332.38	7.86	332.38	260	
270	244.61	10.51	261.54	278.83	295.74	311.71	326.55	340.41	8.03	340.41	270	
280	255.12	11.52	272.36	289.49	305.85	321.12	335.31	348.73	8.32	348.73	280	
290	266.64	12.46	283.89	300.57	316.21	330.73	344.36	357.48	8.75	357.48	290	
300	279.10	13.21	296.00	311.95	326.78	340.63	353.84	363.00	9.37	363.00	300	
310	292.31	13.63	308.50	323.57	337.61	350.93	363.00	370.00	10.25	370.00	310	
320	305.94	13.74	321.19	335.38	348.80	363.00	370.00	377.00	11.43	377.00	320	
330	319.68	13.62	333.97	347.45	359.87	370.00	377.00	384.00	12.94	384.00	330	
340	333.30	13.42	346.80	359.87	370.00	377.00	384.00	391.00	14.68	391.00	340	
350	346.72	13.28	359.78	370.00	377.00	377.00	384.00	391.00	16.38	391.00	350	
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	107.47	17.47	107.47	360	

		$e = 0.20 \quad i = 45^\circ$							
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	10.73	22.21	35.26	50.77	69.25	90.00	0°	
10	10.91	22.41	35.50	51.05	69.58	90.35	111.08	10	
20	22.33	35.41	50.94	69.45	90.21	110.95	129.40	20	
30	34.72	50.11	68.48	89.18	109.98	128.57	144.19	30	
40	48.40	66.46	87.00	107.92	126.80	142.71	156.06	40	
50	63.32	83.56	104.61	123.93	140.32	154.03	165.90	50	
60	78.90	99.99	119.87	136.92	151.17	163.37	174.40	60	
70	94.11	114.56	132.47	147.43	160.12	171.39	182.06	70	
80	108.03	126.89	142.79	156.13	167.77	178.56	189.24	80	
90	120.16	137.17	151.38	163.55	174.57	185.20	196.20	90	
100	130.51	145.80	158.71	170.10	180.81	191.57	203.13	100	
110	139.34	153.20	165.16	176.08	186.72	197.83	210.20	110	
120	146.94	159.69	171.00	181.68	192.47	204.14	217.53	120	
130	153.62	165.53	176.42	187.07	198.21	210.64	225.23	130	
140	159.60	170.92	181.60	192.39	204.05	217.42	233.36	140	
150	165.09	176.01	186.66	197.76	210.12	224.61	241.97	150	
160	170.24	180.94	191.70	203.29	216.53	232.29	251.02	160	
170	175.17	185.81	196.85	209.08	223.38	240.49	260.41	170	
180	180.00	190.73	202.21	215.26	230.77	249.25	270.00	180	
190	184.83	195.80	207.89	221.95	238.79	258.50	279.59	190	
200	189.76	201.14	214.02	229.28	247.49	268.12	288.98	200	
210	194.91	206.88	220.75	237.36	256.86	277.94	298.03	210	
220	200.40	213.16	228.25	246.28	266.81	287.73	306.64	220	
230	206.38	220.17	236.66	256.07	277.12	297.30	314.77	230	
240	213.06	228.13	246.14	266.65	287.58	306.51	322.47	240	
250	220.66	237.25	256.74	277.81	297.92	315.29	329.80	250	
260	229.49	247.74	268.39	289.24	307.93	323.65	336.87	260	
270	239.84	259.68	280.77	300.56	317.50	331.66	343.80	270	
280	251.97	272.88	293.43	311.51	326.64	339.43	350.76	280	
290	265.89	286.85	305.88	321.94	335.41	347.13	357.94	290	
300	281.10	300.85	317.75	331.86	343.98	354.97	5.60	300	
310	296.68	314.25	328.92	341.41	352.58	3.23	14.10	310	
320	311.60	326.71	339.49	350.82	1.50	12.29	23.94	320	
330	325.28	338.26	349.70	0.41	11.15	22.68	35.81	330	
340	337.67	349.16	359.89	10.62	22.09	35.12	50.60	340	
350	349.09	359.82	10.55	22.01	35.03	50.49	68.92	350	
360	0.00	10.73	22.21	35.26	50.77	69.25	90.00	360	

$e = 0.20 \quad i = 60^\circ$										
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°	28.60	
10	7.76	16.26	26.77	41.18	62.23	90.50	118.60	10	20.68	
20	16.19	26.69	41.07	62.06	90.30	118.44	139.28	20	13.69	
30	26.10	40.24	60.85	88.84	117.21	138.44	152.97	30	9.60	
40	38.54	58.37	85.77	114.58	136.62	151.70	162.57	40	7.36	
50	54.60	80.94	110.23	133.57	149.60	161.00	169.93	50	6.10	
60	74.49	103.99	129.08	146.53	158.73	168.07	176.03	60	5.43	
70	91.72	122.88	142.31	155.69	165.65	173.89	181.46	70	5.10	
80	114.81	136.71	151.77	162.62	171.29	178.98	186.56	80	5.05	
90	129.42	146.76	158.90	168.21	176.15	183.69	191.61	90	5.20	
100	140.39	154.34	164.59	172.97	180.57	188.23	196.81	100	5.56	
110	148.73	160.34	169.39	177.22	184.76	192.81	202.37	110	6.14	
120	155.29	165.33	173.61	181.19	188.89	197.59	208.51	120	6.97	
130	160.67	169.66	177.47	185.01	193.10	202.72	215.48	130	8.07	
140	165.27	173.55	181.13	188.83	197.52	208.42	233.55	140	9.47	
150	169.34	177.18	184.72	192.76	202.31	214.90	233.02	150	11.04	
160	173.07	180.67	188.33	196.93	207.64	222.44	244.06	160	12.51	
170	176.58	184.12	192.09	201.47	213.75	231.33	256.57	170	13.43	
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180	13.43	
190	183.42	191.31	200.52	212.44	229.41	253.94	283.43	190	12.51	
200	186.93	195.29	205.52	219.40	239.63	267.34	295.94	200	11.04	
210	190.66	199.72	211.36	227.82	251.74	281.15	306.98	210	9.47	
220	194.73	204.80	218.38	238.14	265.49	294.33	316.45	220	8.07	
230	199.33	210.83	227.06	250.67	280.02	306.13	324.52	230	6.97	
240	204.71	218.27	237.97	265.27	294.14	316.31	331.49	240	6.14	
250	211.27	227.71	251.57	280.98	306.85	325.01	337.63	250	5.56	
260	219.61	239.94	267.72	296.26	317.78	332.51	343.19	260	5.20	
270	230.58	255.55	285.06	309.86	327.06	339.12	348.39	270	5.05	
280	245.28	274.07	301.51	321.38	335.03	345.14	353.44	280	5.10	
290	264.19	293.19	315.65	331.03	342.07	350.82	358.54	290	5.43	
300	285.51	310.19	327.29	339.29	348.52	356.44	363.07	300	6.10	
310	305.40	324.02	336.92	346.62	354.74	362.88	369.10	310	7.36	
320	321.46	335.09	345.18	353.48	361.06	376.70	376.70	320	9.60	
330	333.90	344.23	352.67	362.67	374.94	384.66	384.66	330	13.69	
340	343.81	352.29	359.93	375.55	384.00	392.45	392.45	340	20.68	
350	352.24	359.88	375.55	384.00	392.45	392.45	392.45	350	28.60	
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360		

$e = 0.20 \quad i = 75^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	3.97	8.50	14.51	24.15	46.96	90.00	0°	
10	4.04	8.58	14.63	24.36	44.51	90.97	136.49	10	46.49
20	8.55	14.58	24.28	44.31	90.58	136.29	155.98	20	19.49
30	14.23	23.65	42.87	87.76	134.81	155.35	165.21	30	9.23
40	22.41	40.04	81.86	131.46	153.93	164.43	170.77	40	5.56
50	36.07	72.87	125.45	151.45	163.11	169.89	174.75	50	3.98
60	61.80	115.70	147.49	161.11	168.61	173.76	177.94	60	3.19
70	101.11	141.31	158.20	166.84	172.46	176.83	180.75	70	2.81
80	131.64	154.01	164.47	170.80	175.46	179.47	183.41	80	2.66
90	147.80	161.26	168.70	173.83	178.01	181.91	186.07	90	2.66
100	156.81	166.03	171.88	176.35	180.30	184.28	188.89	100	2.82
110	162.55	169.53	174.46	178.56	182.47	186.71	192.03	110	3.14
120	166.60	172.28	176.68	180.61	184.63	189.32	195.70	120	3.67
130	169.71	174.60	178.69	182.60	186.87	192.23	200.25	130	4.55
140	172.25	176.65	180.59	184.60	189.28	195.65	206.20	140	5.95
150	174.43	178.54	182.45	186.69	191.99	199.86	214.51	150	8.31
160	176.40	180.34	184.34	188.95	195.17	205.33	226.78	160	12.27
170	178.23	182.13	186.33	191.51	199.08	212.89	245.23	170	18.45
180	180.00	183.97	188.50	194.51	204.15	224.01	270.00	180	24.77
190	181.77	185.91	190.96	198.21	211.14	240.92	294.77	190	24.77
200	183.60	188.06	193.88	203.03	221.46	264.87	313.22	200	12.27
210	185.57	190.51	197.51	209.74	237.48	290.85	325.49	210	8.31
220	187.75	193.45	202.30	219.80	261.33	311.14	333.80	220	5.95
230	190.29	197.17	209.09	235.87	288.85	324.66	339.75	230	4.55
240	193.40	202.21	219.60	260.91	310.88	333.69	344.30	240	3.67
250	197.45	209.64	237.23	290.54	325.36	340.08	347.97	250	3.14
260	203.19	221.81	265.60	313.63	334.84	344.93	351.11	260	2.82
270	212.20	243.54	297.46	328.20	341.46	348.83	353.93	270	2.66
280	228.36	277.84	319.82	337.53	346.45	352.18	356.59	280	2.66
290	258.89	309.61	333.16	344.01	350.49	355.22	359.25	290	2.81
300	298.20	328.50	341.61	348.92	354.00	358.15	2.06	300	3.19
310	323.93	339.41	347.56	352.98	357.27	1.18	5.25	310	3.98
320	337.59	346.49	352.20	356.61	0.55	4.56	9.23	320	5.56
330	345.77	351.70	356.19	0.15	4.13	8.70	14.79	330	9.23
340	351.45	355.99	359.96	3.93	8.45	14.44	24.02	340	19.49
350	355.96	359.94	3.90	8.41	14.39	23.93	43.51	350	46.49
360	0.00	3.97	8.50	14.51	24.15	44.01	90.00	360	

		$e = 0.30 \quad i = 15^\circ$								
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	14.51	18.87	29.15	19.17	44.01	19.51	59.13	19.80	74.50
10	18.69	33.38	18.32	48.32	18.63	63.52	18.88	78.93	19.00	94.45
20	36.70	51.70	17.08	66.95	17.30	82.40	17.37	97.93	19.20	113.37
30	53.50	68.78	15.44	84.25	15.50	99.77	15.41	115.20	19.40	130.42
40	68.76	84.22	13.70	99.75	13.61	115.18	13.43	130.40	19.60	145.35
50	82.39	97.92	12.05	113.36	11.91	128.61	11.71	143.60	19.80	158.34
60	94.49	109.97	10.63	125.27	10.46	140.32	10.28	155.11	20.00	169.70
70	105.24	120.60	9.45	135.73	9.28	150.60	9.12	165.24	20.20	179.75
80	114.82	130.05	8.48	145.01	8.33	159.72	8.23	174.27	20.40	188.77
90	123.45	138.53	7.71	153.34	7.60	167.95	7.53	182.45	20.60	196.98
100	131.30	146.24	7.11	160.94	7.02	175.48	6.98	189.98	20.80	204.56
110	138.54	153.35	6.63	167.96	6.56	182.46	6.56	196.99	21.00	211.66
120	145.27	159.98	6.24	174.52	6.20	189.02	6.22	203.59	21.20	218.36
130	151.59	166.22	5.96	180.72	5.96	195.24	5.99	209.88	21.40	224.76
140	157.62	172.18	5.74	186.68	5.75	201.23	5.81	215.96	21.60	230.94
150	163.40	177.92	5.61	192.43	5.63	207.04	5.70	221.86	21.80	236.95
160	169.03	183.53	5.51	198.06	5.55	212.74	5.64	227.66	22.00	242.85
170	174.54	189.04	5.46	203.61	5.54	218.38	5.63	233.40	22.20	248.69
180	180.00	194.51	5.49	209.15	5.56	224.01	5.66	239.13	22.40	254.50
190	185.46	200.00	5.57	214.71	5.66	229.67	5.76	244.90	22.60	260.33
200	190.97	205.57	5.69	220.37	5.79	235.43	5.89	250.74	22.80	266.22
210	196.60	211.26	5.87	226.16	5.97	241.32	6.07	256.71	23.00	272.23
220	202.38	217.13	6.13	232.13	6.24	247.39	6.33	262.85	23.20	278.37
230	208.41	223.26	6.43	238.37	6.55	253.72	6.63	269.22	23.40	284.73
240	214.73	229.69	6.85	244.92	6.95	260.35	7.01	275.87	23.60	291.33
250	221.46	236.54	7.36	251.87	7.45	267.36	7.48	282.87	23.80	298.27
260	228.70	243.90	7.98	259.32	8.05	274.84	8.05	290.32	24.00	305.61
270	236.55	251.88	8.74	267.37	8.77	282.89	8.71	298.28	24.20	313.45
280	245.18	260.62	9.65	276.14	9.63	291.60	9.53	306.88	24.40	321.90
290	254.76	270.27	10.76	285.77	10.67	301.13	10.51	316.25	24.60	331.11
300	265.51	281.03	12.03	296.44	11.87	311.64	11.67	326.58	24.80	341.27
310	277.61	293.06	13.46	308.31	13.24	323.31	13.01	338.05	25.00	352.61
320	291.24	306.52	15.01	321.55	14.75	336.32	14.56	350.90	25.20	359.90
330	306.50	321.53	16.80	336.30	16.30	350.88	16.21	357.16	25.40	366.28
340	323.30	338.04	18.81	352.60	17.74	370.99	17.84	386.39	25.60	373.72
350	341.31	355.85	18.69	369.13	18.81	388.18	19.08	397.22	25.80	397.22
360	0.00	14.51	18.87	29.15	19.17	44.01	19.51	59.13	19.80	74.50

$e = 0.30 \quad i = 30^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	13.06	26.57	40.89	56.31	72.81	90.00	0°	0°
10	16.87	30.57	45.20	60.94	77.69	94.96	112.02	10	10
20	33.76	48.62	64.61	81.54	98.83	115.74	131.71	20	20
30	50.46	66.58	83.59	100.87	117.69	133.53	148.24	30	30
40	66.56	83.56	100.84	117.67	133.51	148.22	161.99	40	40
50	81.53	98.82	115.73	131.70	146.54	160.40	173.63	50	50
60	95.01	112.06	128.27	143.36	157.41	170.74	183.76	60	60
70	106.90	123.41	138.85	153.19	166.71	179.77	192.84	70	70
80	117.28	133.15	147.89	161.67	174.86	187.87	201.15	80	80
90	126.38	141.60	155.77	169.17	182.20	195.31	208.92	90	90
100	134.42	149.07	162.79	175.94	188.96	202.28	216.31	100	100
110	141.61	155.78	169.18	182.21	195.32	208.93	223.43	110	110
120	148.13	161.91	175.09	188.10	201.38	215.36	230.35	120	120
130	154.13	167.60	180.65	193.73	207.26	221.64	237.11	130	130
140	159.74	172.98	185.99	199.20	213.04	227.85	243.79	140	140
150	165.04	178.14	191.18	204.59	218.78	234.03	250.39	150	150
160	170.14	183.16	196.29	209.96	224.54	240.23	256.95	160	160
170	175.10	188.12	201.40	215.38	230.37	246.48	263.48	170	170
180	180.00	193.06	206.57	220.89	236.31	252.81	270.00	180	180
190	184.90	198.08	211.85	226.57	242.41	259.24	276.52	190	190
200	189.86	203.22	217.31	232.45	248.71	265.79	283.05	200	200
210	194.96	208.56	223.03	238.61	255.24	272.48	289.61	210	210
220	200.26	214.17	229.07	245.09	262.03	279.32	296.21	220	220
230	205.87	220.15	235.51	251.96	269.13	286.34	302.89	230	230
240	211.87	226.59	242.43	259.26	276.54	293.54	309.65	240	240
250	218.39	233.61	249.94	267.06	284.30	300.96	316.57	250	250
260	225.58	241.35	258.13	275.40	292.44	308.62	323.69	260	260
270	233.62	249.95	267.07	284.31	300.97	316.58	331.08	270	270
280	242.72	259.56	276.84	293.83	309.92	324.89	338.85	280	280
290	253.10	270.30	287.49	303.97	319.36	333.67	347.16	290	290
300	264.99	282.26	299.02	314.76	329.39	343.09	356.24	300	300
310	278.47	295.40	311.39	326.25	340.13	353.36	363.37	310	310
320	293.44	309.56	324.55	338.53	351.82	358.48	370.41	320	320
330	309.54	324.53	338.52	351.81	4.82	18.00	31.76	330	330
340	326.24	340.12	353.36	363.37	19.59	33.45	48.29	340	340
350	343.13	356.27	366.68	370.41	17.09	51.77	67.98	350	350
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	360	360

$e = 0.30 \quad i = 45^\circ$										
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	10.73	22.21	35.26	50.77	69.25	90.00	0°	26.35	
10	13.91	25.75	39.42	55.76	75.04	96.07	116.35	10	21.15	
20	28.62	42.83	59.84	79.68	100.77	120.56	137.50	20	15.68	
30	44.69	62.06	82.16	103.23	122.73	139.32	153.18	30	11.95	
40	62.03	82.13	103.20	122.70	139.30	153.17	165.13	40	9.66	
50	79.66	100.76	120.55	137.49	151.65	163.79	174.79	50	8.28	
60	96.13	116.40	134.01	148.73	161.24	172.42	183.07	60	7.47	
70	110.41	128.94	144.49	157.58	169.08	179.32	190.54	70	6.99	
80	122.28	138.94	152.87	164.87	175.80	186.44	197.53	80	6.75	
90	132.06	147.10	159.82	171.12	181.80	192.60	204.28	90	6.69	
100	140.20	153.93	165.81	176.69	187.34	198.50	210.97	100	6.73	
110	147.10	159.83	171.13	181.80	192.61	204.29	217.70	110	6.87	
120	153.09	165.06	175.98	186.63	197.73	210.09	224.57	120	7.05	
130	158.40	169.82	180.53	191.28	202.82	215.97	231.62	130	7.29	
140	163.22	174.26	184.90	195.88	207.97	222.05	238.91	140	7.51	
150	167.69	178.48	189.16	200.49	213.26	228.37	246.42	150	7.73	
160	171.92	182.58	193.42	205.21	218.78	234.99	254.15	160	7.89	
170	176.00	186.64	197.75	210.10	224.59	241.94	262.04	170	7.96	
180	180.00	190.73	202.21	215.26	230.77	249.25	270.00	180	7.96	
190	184.00	194.92	206.89	220.77	237.38	256.89	277.96	190	7.89	
200	188.08	199.30	211.89	226.73	244.48	264.85	285.85	200	7.73	
210	192.31	203.96	217.32	233.23	252.12	273.04	293.58	210	7.51	
220	196.78	209.00	223.27	240.37	260.37	281.37	301.09	220	7.29	
230	201.60	214.56	229.92	248.26	268.94	289.76	308.38	230	7.05	
240	206.91	220.79	237.40	256.92	277.99	298.08	315.43	240	6.87	
250	212.90	227.93	245.90	266.40	287.34	306.30	322.30	250	6.73	
260	219.80	236.21	255.56	276.60	296.83	314.38	329.03	260	6.69	
270	227.94	245.92	266.41	287.35	306.31	322.30	335.72	270	6.75	
280	237.72	257.28	278.36	298.41	315.70	330.14	342.47	280	6.99	
290	249.59	270.37	291.10	309.53	324.98	338.00	349.46	290	7.47	
300	263.87	284.91	304.19	320.54	334.22	346.06	356.93	300	8.28	
310	280.34	300.18	317.18	331.39	343.56	354.57	364.57	310	9.66	
320	297.97	315.33	329.83	342.20	353.31	363.95	373.95	320	11.95	
330	315.31	329.82	342.19	353.29	363.29	373.29	382.29	330	15.68	
340	331.38	343.55	354.57	363.29	373.29	382.29	391.29	340	21.15	
350	346.09	356.96	366.96	373.29	382.29	391.29	400.29	350	26.35	
360	0.00	10.73	22.21	35.26	50.77	69.25	90.00	360		

$e = 0.30 \quad i = 60^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°	
10	9.93	18.83	30.17	46.09	69.30	98.55	125.01	10	35.01
20	21.10	33.24	50.58	75.55	105.06	129.86	147.06	20	22.05
30	34.97	53.12	78.98	108.39	132.27	148.71	160.33	30	9.04
40	53.10	78.94	108.36	132.24	148.69	169.32	169.37	40	6.94
50	75.53	105.04	129.85	147.05	159.11	168.38	176.31	50	5.86
60	98.63	125.07	143.80	156.76	166.49	174.62	182.17	60	
70	117.75	138.81	153.23	163.74	172.23	179.87	187.49	70	5.32
80	131.78	148.37	160.08	169.17	177.03	184.56	192.59	80	5.10
90	141.92	155.41	165.43	173.70	181.27	188.98	197.69	90	5.10
100	149.49	160.92	169.86	177.66	185.20	193.31	202.99	100	5.67
110	155.42	165.44	173.70	181.28	188.99	197.70	208.66	110	6.20
120	160.26	169.32	177.16	184.70	192.74	202.28	214.86	120	
130	164.36	172.76	180.38	188.03	196.57	207.17	221.76	130	6.90
140	167.97	175.93	183.47	191.37	200.58	212.53	229.55	140	7.79
150	171.23	178.92	186.51	194.80	204.88	218.50	238.32	150	8.77
160	174.27	181.83	189.58	198.41	209.60	225.27	248.13	160	9.81
170	177.17	184.71	192.75	202.29	214.88	232.99	258.81	170	10.68
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180	11.19
190	182.83	190.67	199.73	211.37	227.85	251.77	281.19	190	11.19
200	185.73	193.91	203.75	216.91	235.98	262.73	291.87	200	10.68
210	188.77	197.45	208.33	223.42	245.48	274.29	301.68	210	9.81
220	192.03	201.40	213.65	231.19	256.38	285.87	310.45	220	8.77
230	195.64	205.97	220.04	240.57	268.50	296.93	318.24	230	7.79
240	199.74	211.39	227.88	251.81	281.23	307.04	325.14	240	6.90
250	204.58	218.07	237.69	264.91	293.82	316.09	331.34	250	6.20
260	210.51	226.58	249.99	279.30	305.58	324.15	337.01	260	5.67
270	218.08	237.70	264.93	293.84	316.11	331.35	342.31	270	5.30
280	228.22	252.29	281.74	307.41	325.40	337.91	347.41	280	5.10
290	242.25	270.52	298.62	319.40	333.64	344.06	352.51	290	5.10
300	261.37	290.63	313.86	329.80	341.14	350.05	357.83	300	5.32
310	284.47	309.43	326.77	338.91	348.22	356.16	369	310	5.86
320	306.90	325.05	337.66	347.21	355.26	364	10.63	320	6.94
330	325.03	337.64	347.20	355.25	2.79	10.62	19.67	330	9.04
340	338.90	348.21	356.15	369	11.61	20.88	32.94	340	13.27
350	350.07	357.85	5.40	13.53	23.27	36.24	54.99	350	22.05
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360	35.01

$e = 0.30 \quad i = 75^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	3.97	6.04	8.50	14.51	24.15	29.72	0.00	0°
10	5.18	10.01	8.73	16.75	28.27	53.87	106.20	53.53	10
20	11.30	18.74	15.87	32.20	63.54	117.46	148.20	161.46	20
30	19.90	34.61	34.71	69.39	122.72	150.34	182.53	169.52	30
40	34.58	69.32	48.11	122.66	150.32	162.52	169.51	174.45	40
50	63.51	117.43	26.16	148.19	161.45	168.83	173.93	178.09	50
60	106.34	143.59	12.04	159.25	167.47	172.91	177.21	181.13	60
70	135.47	155.63	6.69	165.36	171.41	175.96	179.93	183.90	70
80	149.91	162.32	4.36	169.38	174.35	178.46	182.37	186.59	80
90	157.92	166.68	3.17	172.34	176.73	180.66	184.68	189.38	90
100	163.04	169.85	2.49	174.71	178.79	182.70	186.98	192.39	100
110	166.68	172.34	2.08	176.73	180.66	184.68	189.38	195.80	110
120	169.48	174.42	1.82	178.53	182.43	186.67	191.97	199.83	120
130	171.75	176.24	1.65	180.19	184.18	188.75	194.88	204.80	130
140	173.70	177.89	1.55	181.80	185.94	191.00	198.27	211.26	140
150	175.43	179.44	1.51	183.38	187.79	193.50	202.38	219.99	150
160	177.03	180.95	1.49	184.99	189.78	196.39	207.59	232.20	160
170	178.53	182.44	1.53	186.68	191.98	199.84	214.48	249.08	170
180	180.00	183.97	1.60	188.50	194.51	204.15	224.01	270.00	180
190	181.47	185.57	1.74	190.52	197.52	209.76	237.54	290.92	190
200	182.97	187.31	1.93	192.83	201.25	217.48	256.15	307.80	200
210	184.57	189.24	2.23	195.59	206.10	228.61	278.25	320.01	210
220	186.30	191.47	2.68	199.01	212.76	244.91	298.78	328.74	220
230	188.25	194.15	3.38	203.51	222.54	267.10	314.46	335.20	230
240	190.52	197.53	4.54	209.79	237.59	290.99	325.55	340.17	240
250	193.32	202.07	6.61	219.30	260.24	310.46	333.52	344.20	250
260	196.96	208.68	10.63	234.87	287.55	324.11	339.49	347.61	260
270	202.08	219.31	19.02	260.27	310.49	333.53	344.21	350.62	270
280	210.09	238.33	32.67	291.87	325.91	340.35	348.13	353.41	280
290	224.53	271.00	35.03	316.51	336.08	345.61	351.59	356.10	290
300	253.66	306.03	306.03	331.69	343.23	349.97	354.81	358.87	300
310	296.49	327.81	21.78	341.27	348.71	353.84	358.01	1.91	310
320	325.42	340.11	12.30	347.99	353.30	357.54	1.45	5.55	320
330	340.10	347.98	5.85	353.29	357.54	1.44	5.55	10.48	330
340	348.70	353.83	5.06	358.01	1.91	6.07	11.17	18.54	340
350	354.82	358.89	5.08	2.80	7.10	12.55	20.78	36.47	350
360	0.00	3.97	8.50	14.51	24.15	44.01	90.00	53.53	360

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	14.51	24.52	29.15	33.00	36.85	40.70	0°
10	24.25	39.03	22.76	54.06	23.13	25.34	27.57	10
20	22.37	61.79	19.69	77.19	23.35	23.35	22.83	20
30	46.62	19.42	19.82	92.70	19.76	15.82	18.88	30
40	66.04	16.30	16.30	112.46	12.73	15.56	15.37	40
50	82.34	13.58	13.36	128.56	14.35	12.91	12.68	50
60	95.92	11.39	11.07	141.69	15.64	10.73	10.68	60
70	107.31	9.69	9.37	152.57	16.71	9.14	9.21	70
80	117.00	8.40	8.12	161.80	17.63	8.02	8.13	80
90	125.40	7.42	7.18	169.82	18.32	7.12	7.30	90
100	132.82	6.65	6.54	176.94	19.14	6.46	6.65	100
110	139.47	6.08	5.99	183.37	19.70	5.98	6.18	110
120	145.55	5.63	5.55	189.30	20.38	5.59	5.79	120
130	151.18	5.27	5.21	194.83	20.97	5.30	5.49	130
140	156.45	5.00	4.96	200.06	21.47	5.09	5.26	140
150	161.45	4.80	4.77	205.07	21.86	4.93	5.08	150
160	166.25	4.66	4.65	209.91	22.19	4.82	4.96	160
170	170.91	4.57	4.58	214.66	22.61	4.78	4.84	170
180	175.48	4.52	4.53	219.34	23.03	4.74	4.84	180
190	180.00	4.52	4.55	224.01	23.43	4.77	4.84	190
200	184.52	4.57	4.61	228.70	23.80	4.85	4.90	200
210	189.09	4.66	4.71	233.46	24.17	4.94	4.96	210
220	193.75	4.80	4.86	238.34	24.53	5.10	5.08	220
230	198.55	5.00	5.08	243.37	24.89	5.30	5.26	230
240	203.55	5.27	5.36	248.62	25.25	5.57	5.49	240
250	208.82	5.63	5.72	254.16	25.61	5.88	5.79	250
260	214.45	6.08	6.19	260.05	25.97	6.27	6.18	260
270	220.53	6.65	6.77	266.39	26.34	6.78	6.65	270
280	227.18	7.42	7.54	273.28	26.72	7.42	7.30	280
290	234.60	8.40	8.52	280.89	27.11	8.27	8.13	290
300	243.00	9.69	9.77	289.41	27.51	9.35	9.21	300
310	252.69	11.39	11.41	299.09	27.92	10.80	10.68	310
320	264.08	13.58	13.51	310.26	28.34	12.73	12.68	320
330	277.66	16.30	16.09	323.36	28.77	15.29	15.37	330
340	293.96	19.42	19.08	338.91	29.20	18.64	18.88	340
350	313.38	22.37	22.05	357.48	29.63	22.43	22.83	350
360	335.75	24.25	24.18	381.97	30.06	25.48	25.77	360

$e = 0.40 \quad i = 30^\circ$										
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	13.06	26.57	40.89	56.31	72.81	90.00	0°	28.30	
10	21.99	36.01	51.04	67.20	84.23	101.51	118.30	10	23.37	
20	43.49	59.11	75.77	93.01	110.13	126.45	141.67	20	17.94	
30	63.64	80.52	97.81	114.76	130.79	145.70	159.61	30	13.97	
40	81.47	98.76	115.67	131.64	146.49	160.36	173.58	40	11.38	
50	96.60	113.59	129.70	144.69	158.66	171.95	184.96	50	9.65	
60	109.16	125.55	140.83	155.05	168.47	181.52	194.61	60	8.47	
70	119.60	135.31	149.90	163.57	176.71	189.73	203.08	70	7.65	
80	128.40	143.48	157.53	170.86	183.87	197.02	210.73	80	7.05	
90	135.94	150.49	164.13	177.25	190.28	203.65	217.78	90		
100	142.53	156.64	170.00	183.02	196.15	209.81	224.38	100	6.60	
110	148.41	162.17	175.34	188.35	201.65	215.64	230.65	110	6.27	
120	153.74	167.23	180.29	193.35	206.87	221.22	236.66	120	6.01	
130	158.65	171.94	184.95	198.13	211.90	226.50	242.48	130	5.82	
140	163.25	176.40	189.42	202.75	216.82	231.92	248.14	140	5.66	
150	167.62	180.68	193.75	207.29	221.67	237.14	253.69	150	5.55	
160	171.83	184.84	198.02	211.79	226.50	242.34	259.17	160	5.48	
170	175.94	188.96	202.28	216.31	231.38	247.56	264.60	170	5.43	
180	180.00	193.06	206.57	220.89	236.31	252.81	270.00	180	5.40	
190	184.06	197.21	210.93	225.58	241.35	258.13	275.40	190	5.40	
200	188.17	201.46	215.44	230.43	246.55	263.55	280.83	200	5.48	
210	192.38	205.85	220.12	235.48	251.93	269.10	286.31	210	5.55	
220	196.75	210.44	225.05	240.79	257.53	274.80	291.86	220	5.66	
230	201.35	215.32	230.31	246.42	263.41	280.70	297.52	230	5.82	
240	206.26	220.57	235.96	252.44	269.62	286.82	303.34	240	6.01	
250	211.59	226.29	242.11	258.93	276.21	293.22	309.35	250	6.27	
260	217.47	232.62	248.89	265.97	283.23	299.94	315.62	260	6.60	
270	224.06	239.71	256.40	273.66	290.76	307.04	322.22	270	7.05	
280	231.60	247.79	264.84	282.11	298.88	314.63	329.27	280	7.65	
290	240.40	257.12	274.39	291.46	307.70	322.83	336.92	290	8.47	
300	250.84	267.98	285.21	301.82	317.37	331.81	345.39	300		
310	263.40	280.68	297.51	313.36	328.09	341.86	355.04	310	9.65	
320	278.53	295.45	311.44	326.30	340.18	353.41	6.42	320	11.38	
330	296.36	312.29	327.09	340.92	354.13	7.14	20.39	330	13.97	
340	316.51	331.01	344.63	357.74	10.77	24.17	38.33	340	17.94	
350	338.01	351.32	4.33	17.50	31.23	21.73	61.70	350	23.37	
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	360	28.30	

$e = 0.40$		$i = 45^\circ$														
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	10.73	19.95	22.21	23.07	35.26	50.77	32.18	69.25	34.75	90.00	33.40	123.40	132.14	147.16	159.88
10	18.25	30.68	45.28	62.76	82.95	104.00	123.40	33.40	104.00	28.14	123.40	132.14	147.16	159.88	174.75	192.01
20	37.76	53.77	72.74	93.69	114.17	132.14	147.16	159.88	174.75	18.74	159.88	174.75	187.97	205.89	224.88	243.83
30	58.74	78.44	99.53	119.46	136.58	150.88	163.12	174.75	187.97	12.87	163.12	174.75	187.97	205.89	224.88	243.83
40	79.59	100.69	120.48	137.44	151.60	163.75	174.75	187.97	192.01	6.73	174.75	187.97	205.89	224.88	243.83	263.40
50	98.06	118.14	135.48	149.96	162.31	173.41	184.05	192.01	205.89	5.64	184.05	192.01	205.89	224.88	243.83	263.40
60	113.06	131.19	146.37	159.20	170.55	181.24	192.01	205.89	224.88	5.39	192.01	205.89	224.88	243.83	263.40	270.00
70	124.83	141.07	154.67	166.47	177.31	187.97	199.68	212.33	231.14	5.25	199.68	212.33	231.14	243.83	263.40	270.00
80	134.15	148.84	161.34	172.51	183.16	194.04	205.07	218.62	237.45	5.24	205.07	218.62	237.45	243.83	263.40	270.00
90	141.69	155.19	166.93	177.76	188.43	199.68	212.33	231.14	243.83	5.48	212.33	231.14	243.83	263.40	270.00	270.00
100	147.96	160.57	171.81	182.47	193.30	205.07	218.62	237.45	243.83	5.65	218.62	237.45	243.83	263.40	270.00	270.00
110	153.34	165.28	176.19	186.83	197.95	210.34	224.88	243.83	263.40	6.08	224.88	243.83	263.40	270.00	270.00	270.00
120	158.06	169.51	180.23	190.97	202.48	215.58	231.14	243.83	263.40	6.31	231.14	243.83	263.40	270.00	270.00	270.00
130	162.30	173.40	184.04	194.97	206.94	220.83	237.45	243.83	263.40	6.56	237.45	243.83	263.40	270.00	270.00	270.00
140	166.20	177.06	187.71	198.90	211.43	226.18	243.17	263.40	270.00	6.78	243.17	263.40	270.00	270.00	270.00	270.00
150	169.84	180.55	191.30	202.84	216.00	231.66	249.25	263.40	270.00	6.97	249.25	263.40	270.00	270.00	270.00	270.00
160	173.32	183.96	194.88	206.84	220.71	237.31	255.56	263.40	270.00	7.16	255.56	263.40	270.00	270.00	270.00	270.00
170	176.69	187.34	198.50	210.97	225.62	243.17	263.40	270.00	270.00	7.29	263.40	270.00	270.00	270.00	270.00	270.00
180	180.00	190.73	202.01	215.26	230.77	249.25	263.40	270.00	270.00	7.48	270.00	270.00	270.00	270.00	270.00	270.00
190	183.31	194.19	206.07	219.80	236.21	255.56	263.40	270.00	270.00	7.55	276.60	270.00	270.00	270.00	270.00	270.00
200	186.68	197.79	210.16	224.66	242.02	262.12	270.00	270.00	270.00	7.66	283.19	270.00	270.00	270.00	270.00	270.00
210	190.16	201.58	214.53	229.89	248.22	268.90	270.00	270.00	270.00	7.71	289.72	270.00	270.00	270.00	270.00	270.00
220	193.80	205.63	219.28	235.59	254.85	275.87	270.00	270.00	270.00	7.83	296.17	270.00	270.00	270.00	270.00	270.00
230	197.70	210.05	224.53	241.87	261.95	283.03	270.00	270.00	270.00	8.13	302.55	270.00	270.00	270.00	270.00	270.00
240	201.94	214.95	230.40	248.81	269.53	290.32	270.00	270.00	270.00	8.24	308.86	270.00	270.00	270.00	270.00	270.00
250	206.66	220.50	237.05	256.52	277.59	297.72	270.00	270.00	270.00	8.31	315.12	270.00	270.00	270.00	270.00	270.00
260	212.04	226.90	244.69	265.07	286.07	305.20	270.00	270.00	270.00	8.44	321.38	270.00	270.00	270.00	270.00	270.00
270	218.31	234.42	253.50	274.48	294.90	312.75	270.00	270.00	270.00	8.58	327.67	270.00	270.00	270.00	270.00	270.00
280	225.85	243.44	263.69	284.73	304.04	320.41	270.00	270.00	270.00	8.66	334.11	270.00	270.00	270.00	270.00	270.00
290	235.17	254.36	275.37	295.71	313.43	328.24	270.00	270.00	270.00	8.71	340.82	270.00	270.00	270.00	270.00	270.00
300	246.94	267.53	288.42	307.23	323.07	336.37	270.00	270.00	270.00	8.83	347.99	270.00	270.00	270.00	270.00	270.00
310	261.94	283.01	302.54	319.16	333.05	345.03	270.00	270.00	270.00	8.98	355.95	270.00	270.00	270.00	270.00	270.00
320	280.41	300.24	317.24	331.43	343.60	354.61	270.00	270.00	270.00	9.13	363.30	270.00	270.00	270.00	270.00	270.00
330	301.26	318.09	332.15	344.23	355.20	366.88	270.00	270.00	270.00	9.28	374.88	270.00	270.00	270.00	270.00	270.00
340	322.24	335.66	347.35	358.15	368.83	379.51	270.00	270.00	270.00	9.41	388.44	270.00	270.00	270.00	270.00	270.00
350	341.75	352.89	363.54	373.54	383.54	393.54	270.00	270.00	270.00	9.58	399.00	270.00	270.00	270.00	270.00	270.00
360	0.00	10.73	22.21	35.26	50.77	69.25	90.00	90.00	90.00	9.71	409.00	270.00	270.00	270.00	270.00	270.00

		$e = 0.40 \quad i = 60^\circ$							
M	M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°
10	13.12	22.76	15.13	35.53	53.95	80.08	109.43	133.00	10
20	28.71	20.65	43.98	66.28	95.21	122.41	141.99	155.47	20
30	49.36	26.08	73.87	103.36	128.62	146.21	158.50	167.89	30
40	75.44	25.89	104.94	129.78	147.01	159.08	176.36	176.28	40
50	101.33	19.72	127.11	145.19	157.76	167.29	175.33	182.87	50
60	121.05	13.49	141.06	154.81	164.96	173.29	180.88	188.56	60
70	134.54	9.39	150.27	161.49	170.34	178.10	185.65	193.82	70
80	143.93	6.88	156.85	166.57	174.69	182.24	190.02	198.95	80
90	150.81	5.32	161.90	170.68	178.41	185.98	194.19	204.11	90
100	156.13	4.32	166.00	174.19	181.75	189.49	198.31	209.46	100
110	160.45	3.65	169.48	177.30	184.84	192.91	202.49	215.15	110
120	164.10	3.18	172.54	180.17	187.80	196.31	206.83	221.27	120
130	167.28	2.86	175.32	182.86	190.70	199.77	211.43	227.93	130
140	170.14	2.64	177.92	185.47	193.61	203.37	216.38	235.20	140
150	172.78	2.48	180.39	188.04	196.59	207.19	221.79	243.12	150
160	175.26	2.40	182.80	190.64	199.69	211.32	227.77	251.66	160
170	177.66	2.34	185.20	193.31	202.99	215.85	234.42	260.70	170
180	180.00	2.34	187.63	196.10	206.57	220.89	241.81	270.00	180
190	182.34	2.40	190.14	199.08	210.51	226.58	249.99	279.30	190
200	184.74	2.48	192.78	202.34	214.94	233.08	258.92	288.34	200
210	187.22	2.64	195.62	205.95	220.01	240.53	268.44	296.88	210
220	189.86	2.86	198.74	210.05	225.92	249.04	278.28	304.80	220
230	192.72	3.18	202.25	214.82	232.91	258.69	288.12	312.07	230
240	195.90	3.65	206.30	220.52	241.27	269.34	297.64	318.73	240
250	199.55	4.32	211.13	227.49	251.27	280.67	306.61	324.85	250
260	203.87	5.32	217.08	236.23	263.05	292.16	314.93	330.54	260
270	209.19	6.88	224.67	247.27	276.32	303.28	322.59	335.89	270
280	216.07	9.39	234.74	261.11	290.39	313.69	329.68	341.05	280
290	225.46	13.49	248.40	277.57	304.25	323.24	336.36	346.18	290
300	238.95	19.72	266.50	295.22	317.06	332.01	342.81	351.44	300
310	258.67	25.89	288.10	312.06	328.56	340.22	349.29	357.13	310
320	284.56	26.08	309.50	326.82	338.94	348.24	356.18	372	320
330	310.64	20.65	327.60	339.51	348.71	356.60	4.14	12.11	330
340	331.29	15.59	342.26	350.98	358.69	6.27	14.52	24.53	340
350	346.88	13.12	354.96	2.50	10.31	19.30	30.79	47.00	350
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	121.11	360

$e = 0.40 \quad i = 75^\circ$											
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M			
0°	0.00	3.97	8.50	14.51	24.15	44.01	90.00	0°	60.97	90.00	0°
10	6.88	12.25	20.29	35.42	71.32	124.27	150.97	10	15.74	150.97	10
20	15.83	26.54	49.68	99.99	140.81	157.97	166.71	20	6.95	166.71	20
30	31.09	60.80	114.64	147.05	160.90	168.48	173.66	30	4.41	173.66	30
40	63.35	117.27	148.13	161.42	168.81	173.91	178.07	40	3.42	178.07	40
50	111.16	145.62	160.21	168.05	173.34	177.58	181.49	50	2.96	181.49	50
60	139.31	157.30	166.32	172.08	176.51	180.45	184.45	60		184.45	60
70	152.26	163.53	170.17	174.96	179.02	182.93	187.26	70	2.81	187.26	70
80	159.34	167.52	172.95	177.25	181.16	185.23	190.08	80	2.82	190.08	80
90	163.87	170.40	175.14	179.18	183.10	187.46	193.04	90	2.96	193.04	90
100	167.10	172.64	176.98	180.90	184.95	189.72	196.30	100	3.26	196.30	100
110	169.59	174.51	178.60	182.51	186.76	192.09	200.03	110	3.73	200.03	110
120	171.61	176.12	180.09	184.06	188.61	194.67	204.43	120	4.40	204.43	120
130	173.34	177.58	181.48	185.59	190.54	197.55	209.83	130	5.40	209.83	130
140	174.86	178.92	182.84	187.14	192.61	200.88	216.68	140	6.85	216.68	140
150	176.25	180.20	184.18	188.76	194.89	204.83	225.61	150	8.93	225.61	150
160	177.54	181.45	185.55	190.49	197.48	209.70	237.37	160	11.76	237.37	160
170	178.79	182.70	186.98	192.39	200.51	215.89	252.45	170	15.08	252.45	170
180	180.00	183.97	188.50	194.51	204.15	224.01	270.00	180	17.55	270.00	180
190	181.21	185.29	190.15	196.96	208.68	234.87	287.55	190	17.55	287.55	190
200	182.46	186.70	192.01	199.88	214.56	249.29	302.63	200	15.08	302.63	200
210	183.75	188.24	194.14	203.49	222.49	266.99	314.39	210	11.76	314.39	210
220	185.14	189.96	196.67	208.12	233.50	285.70	323.32	220	8.93	323.32	220
230	186.66	191.96	199.80	214.40	248.88	302.29	330.17	230	6.85	330.17	230
240	188.39	194.35	203.86	223.35	268.73	315.33	335.57	240	5.40	335.57	240
250	190.41	197.36	209.46	236.78	289.99	325.14	339.97	250	4.40	339.97	250
260	192.90	201.36	217.74	256.74	308.19	332.58	343.70	260	3.73	343.70	260
270	196.13	207.10	231.02	282.08	321.74	338.40	346.96	270	3.26	346.96	270
280	200.66	216.21	253.19	305.69	331.55	343.16	349.92	280	2.96	349.92	280
290	207.74	232.58	284.39	322.76	338.86	347.23	352.74	290	2.82	352.74	290
300	220.69	263.27	312.30	334.28	344.62	350.90	355.55	300	2.81	355.55	300
310	243.84	302.26	330.15	342.44	349.46	354.51	358.51	310	2.96	358.51	310
320	296.65	327.87	341.30	348.73	353.85	358.02	1.93	320	3.42	1.93	320
330	328.91	341.81	349.05	354.10	358.24	2.14	6.34	330	4.41	6.34	330
340	344.17	350.60	355.30	359.32	3.25	7.64	13.29	340		13.29	340
350	353.12	357.39	1.30	5.38	10.27	17.14	29.03	350	6.95	29.03	350
360	0.00	3.97	8.50	14.51	24.15	44.01	90.00	360	15.74	90.00	360

$e = 0.50 \quad i = 15^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	14.51	32.84	29.15	33.39	44.01	33.93	59.13	34.33
10	32.43	47.35	27.93	62.54	28.25	77.94	28.35	93.46	28.18
20	27.48	21.20	21.35	90.79	21.30	106.29	21.07	121.64	20.73
30	16.09	96.63	16.03	112.09	15.83	127.36	15.56	142.37	15.30
40	12.47	112.66	12.32	127.92	12.11	142.92	11.90	157.67	11.74
50	10.00	124.98	9.83	140.03	9.66	154.82	9.52	169.41	9.45
60	8.27	134.81	8.13	149.69	8.00	164.34	7.91	178.86	7.88
70	7.04	142.94	6.92	157.69	6.82	172.25	6.77	186.74	6.79
80	6.17	149.86	6.07	164.51	6.00	179.02	5.98	193.53	6.02
90	5.51	155.93	5.42	170.51	5.37	185.00	5.38	199.55	5.42
100	5.00	161.35	4.93	175.88	4.91	190.38	4.93	204.97	4.98
110	4.62	166.28	4.58	180.79	4.56	195.31	4.59	209.95	4.66
120	4.34	170.86	4.30	185.35	4.31	199.90	4.34	214.61	4.41
130	4.11	175.16	4.09	189.66	4.09	204.24	4.14	219.02	4.21
140	3.95	179.25	3.93	193.75	3.96	208.38	4.01	223.23	4.07
150	3.84	183.18	3.83	197.71	3.86	212.39	3.92	227.30	3.99
160	3.75	187.01	3.76	201.57	3.79	216.31	3.85	231.29	3.93
170	3.73	190.77	3.74	205.36	3.79	220.16	3.85	235.22	3.91
180	3.73	194.51	3.75	209.15	3.80	224.01	3.86	239.13	3.94
190	3.75	198.26	3.78	212.95	3.83	227.87	3.91	243.07	3.97
200	3.84	202.04	3.88	216.78	3.95	231.78	4.01	247.04	4.07
210	3.95	205.92	3.99	220.73	4.06	235.79	4.14	251.11	4.19
220	4.11	209.91	4.17	224.79	4.24	239.93	4.31	255.30	4.37
230	4.34	214.08	4.41	229.03	4.49	244.24	4.56	259.67	4.60
240	4.62	218.49	4.70	233.52	4.78	248.80	4.85	264.27	4.88
250	5.00	223.19	5.09	238.30	5.17	253.65	5.26	269.15	5.26
260	5.51	228.28	5.60	243.47	5.70	258.89	5.75	274.41	5.75
270	6.17	233.88	6.28	249.17	6.37	264.64	6.41	280.16	6.38
280	7.04	240.16	7.16	255.54	7.24	271.05	7.25	286.54	7.20
290	8.27	247.32	8.39	262.78	8.44	278.30	8.42	293.74	8.33
300	10.00	255.71	10.10	271.22	10.11	286.72	10.02	302.07	9.87
310	12.47	265.81	12.51	281.33	12.43	296.74	12.27	311.94	12.05
320	16.09	278.32	16.01	293.76	15.80	309.01	15.52	323.99	15.27
330	278.89	294.33	20.90	309.56	20.55	324.53	20.22	339.26	20.01
340	300.09	315.23	27.01	330.11	26.66	344.75	26.52	359.27	26.60
350	327.57	342.24	32.27	356.77	32.38	11.27	32.74	25.87	33.26
360	0.00	14.51	29.15	29.15	44.01	59.13	74.50	90.00	90.00

$e = 0.50$		$i = 30^\circ$							M	
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M		
0°	0.00	13.06	26.57	33.33	40.89	35.71	37.55	38.14	90.00	
10	29.67	44.23	59.90	76.60	93.86	110.95	127.22	127.22	37.22	
20	57.12	73.67	90.88	108.05	124.50	139.86	154.14	154.14	26.92	
30	80.11	97.39	114.36	130.42	145.35	159.28	172.55	172.55	18.41	
40	98.02	114.97	130.99	145.88	159.78	173.03	186.04	186.04	13.49	
50	111.74	127.97	143.07	157.15	170.49	183.51	196.65	196.65	10.61	
60	122.43	137.94	152.34	165.89	178.98	192.03	205.48	205.48	8.83	
70	131.01	145.90	159.80	173.04	186.05	199.27	213.11	213.11	7.63	
80	138.10	152.50	166.05	179.13	192.18	205.63	219.90	219.90	6.79	
90	144.17	158.17	171.47	184.49	197.66	211.40	226.09	226.09	6.19	
100	149.46	163.16	176.31	189.33	202.66	216.72	231.82	231.82	5.73	
110	154.19	167.66	180.71	193.79	207.32	221.71	237.19	237.19	5.37	
120	158.50	171.79	184.80	197.98	211.74	226.45	242.29	242.29	5.10	
130	162.50	175.66	188.67	201.98	216.00	231.03	247.19	247.19	4.90	
140	166.25	179.32	192.38	205.85	220.12	235.48	251.93	251.93	4.74	
150	169.82	182.85	195.97	209.63	224.18	239.84	256.54	256.54	4.61	
160	173.29	186.30	199.52	213.38	228.21	244.17	261.08	261.08	4.54	
170	176.66	189.68	203.03	217.11	232.23	248.47	265.55	265.55	4.47	
180	180.00	193.06	206.57	220.89	236.31	252.81	270.00	270.00	4.45	
190	183.34	196.48	210.16	224.75	240.46	257.19	274.45	274.45	4.47	
200	186.71	199.95	213.83	228.70	244.70	261.63	278.92	278.92	4.54	
210	190.18	203.55	217.66	232.83	249.11	266.20	283.46	283.46	4.61	
220	193.75	207.29	221.67	237.14	253.69	270.90	288.07	288.07	4.74	
230	197.50	211.24	225.92	241.71	258.50	275.78	292.81	292.81	4.90	
240	201.50	215.48	230.48	246.60	263.61	280.89	297.71	297.71	5.10	
250	205.81	220.08	235.44	251.88	269.05	286.26	302.81	302.81	5.37	
260	210.54	225.16	240.89	257.65	274.92	291.97	308.18	308.18	5.73	
270	215.83	230.86	247.00	264.02	281.30	298.10	313.91	313.91	6.19	
280	221.90	237.39	253.95	271.17	286.33	304.76	320.10	320.10	6.79	
290	228.99	245.01	261.95	279.24	296.13	312.08	326.89	326.89	7.63	
300	237.57	254.14	271.36	288.52	304.94	320.27	334.52	334.52	8.83	
310	248.26	265.33	282.59	299.33	315.06	329.66	343.35	343.35	10.61	
320	261.98	279.26	296.16	312.10	326.91	340.75	353.96	353.96	13.49	
330	279.89	296.76	312.66	327.43	341.24	354.44	7.45	7.45	18.41	
340	302.88	318.35	332.73	346.27	359.34	12.40	25.86	25.86	26.92	
350	330.33	343.98	357.11	10.13	23.50	37.62	52.78	52.78	37.22	
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	90.00		

$e = 0.50 \quad i = 45^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	10.73	22.21	35.26	50.77	69.25	90.00	0°	
10	24.95	27.75	32.41	38.47	43.95	45.87	132.93	10	42.93
20	26.68	31.78	36.45	44.03	49.51	51.43	158.40	20	25.47
30	26.31	28.77	33.71	41.06	46.54	48.46	173.90	30	15.50
40	21.86	20.66	27.77	33.21	39.24	41.16	184.93	40	11.03
50	16.23	14.01	21.69	25.05	31.09	32.91	193.72	50	8.79
60	11.86	9.92	15.83	18.41	23.71	25.53	201.26	60	7.54
70	8.91	7.44	13.28	14.31	19.17	20.99	208.03	70	6.77
80	6.98	5.91	10.53	10.99	15.43	17.25	214.32	80	5.98
90	5.70	4.92	8.92	9.37	13.86	15.28	220.30	90	5.77
100	4.80	4.23	7.68	8.07	12.41	13.86	226.07	100	5.63
110	4.17	3.75	6.87	7.13	11.58	12.93	231.70	110	5.55
120	3.72	3.41	6.34	6.54	11.06	12.41	237.25	120	5.50
130	3.39	3.17	5.98	6.10	10.67	12.02	242.75	130	5.47
140	3.14	3.00	5.66	5.78	10.38	11.73	248.22	140	5.45
150	2.96	2.88	5.42	5.52	10.14	11.49	253.67	150	5.44
160	2.85	2.82	5.22	5.31	9.94	11.29	259.11	160	5.45
170	2.76	2.78	5.04	5.13	9.76	11.10	264.56	170	5.44
180	2.73	2.80	4.91	5.00	9.61	11.00	270.00	180	5.44
190	2.73	2.85	4.81	4.91	9.51	10.90	275.44	190	5.45
200	2.85	3.08	4.78	4.88	9.44	10.83	280.89	200	5.44
210	2.96	3.25	4.80	4.95	9.40	10.79	286.33	210	5.45
220	3.14	3.51	4.93	5.13	9.38	10.75	291.78	220	5.47
230	3.39	3.85	5.14	5.47	9.38	10.71	297.25	230	5.50
240	3.72	4.30	5.48	5.83	9.40	10.67	302.75	240	5.55
250	4.17	4.88	5.86	6.22	9.44	10.63	308.30	250	5.63
260	4.80	5.71	6.83	7.21	9.49	10.59	313.93	260	5.77
270	5.70	6.83	8.05	8.47	9.56	10.56	319.70	270	5.98
280	6.98	8.36	9.59	9.94	9.64	10.54	325.68	280	6.29
290	8.91	10.54	11.49	11.86	9.73	10.52	331.97	290	6.77
300	11.86	13.46	13.63	14.03	9.82	10.50	338.74	300	7.54
310	16.23	17.02	15.73	16.13	9.91	10.48	346.28	310	8.79
320	20.60	20.40	17.43	17.83	10.00	10.46	355.07	320	11.03
330	21.86	22.32	18.46	18.86	10.09	10.44	6.10	330	15.50
340	26.31	22.79	20.46	20.86	10.18	10.42	21.60	340	25.47
350	33.05	23.92	24.57	24.97	10.27	10.40	47.07	350	42.93
360	0.00	10.73	22.21	35.26	50.77	69.25	90.00	360	

$e = 0.50$		$i = 60^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M			
0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°	52.76	61.73	90.00
10	18.21	29.34	44.88	67.57	96.66	123.54	142.76	10	21.60	30.50	142.76
20	41.77	63.09	91.52	119.44	139.97	154.04	164.36	20	11.32	13.64	164.36
30	73.19	102.66	128.10	145.87	158.25	167.68	175.68	30	7.81	8.28	175.68
40	103.72	128.88	146.40	158.63	168.00	175.96	183.49	40	6.31	6.07	183.49
50	124.63	143.50	156.55	166.32	174.47	182.03	189.80	50	5.58	4.98	189.80
60	137.74	152.48	163.17	171.75	179.41	187.01	195.38	60			195.38
70	146.42	158.65	168.01	175.97	183.50	191.41	200.63	70	5.25	4.40	200.63
80	152.62	163.27	171.84	179.50	187.10	195.48	205.77	80	5.18	3.94	205.77
90	157.37	166.98	175.05	182.60	190.41	199.42	210.95	90	5.33	3.88	210.95
100	161.19	170.09	177.87	185.42	193.55	203.30	216.28	100	5.56	3.93	216.28
110	164.40	172.80	180.41	188.06	196.61	207.23	221.84	110	5.87	4.05	221.84
120	167.19	175.24	182.78	190.61	199.66	211.28	227.71	120	6.22	4.24	227.71
130	169.68	177.49	185.03	193.12	202.75	215.52	233.93	130	6.60	4.49	233.93
140	171.96	179.61	187.22	195.62	205.95	220.01	240.53	140	6.95	4.81	240.53
150	174.08	181.65	189.38	198.18	209.29	224.82	247.48	150	7.31	5.21	247.48
160	176.11	183.64	191.57	200.82	212.86	230.03	254.79	160	7.52	5.63	254.79
170	178.07	185.62	193.79	203.60	216.69	235.66	262.31	170	7.69	6.15	262.31
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180			270.00
190	181.93	189.69	198.55	209.78	225.53	248.50	277.69	190	7.69	6.69	277.69
200	183.89	191.84	201.16	213.32	230.70	255.71	285.21	200	7.52	7.21	285.21
210	185.92	194.12	204.02	217.29	236.53	263.44	292.52	210	6.95	8.12	292.52
220	188.04	196.59	207.19	221.79	243.12	271.56	299.47	220	6.60	8.38	299.47
230	190.32	199.30	210.80	227.01	250.59	279.94	306.07	230	6.22	8.49	306.07
240	192.81	202.37	214.99	233.15	259.02	288.43	312.29	240	5.87	8.38	312.29
250	195.60	205.91	219.97	240.46	268.36	296.81	318.16	250	5.56	8.14	318.16
260	198.81	210.13	226.04	249.23	278.47	304.95	323.72	260	5.33	7.82	323.72
270	202.63	215.35	233.68	259.72	289.09	312.77	329.05	270	5.18	7.48	329.05
280	207.38	222.06	243.51	272.02	299.85	320.25	334.23	280	5.14	7.15	334.23
290	213.58	231.09	256.24	285.74	310.36	327.40	339.37	290	5.25	6.97	339.37
300	222.26	243.80	272.36	300.13	320.43	334.37	344.62	300	5.58	6.96	344.62
310	235.37	261.94	291.15	314.23	330.05	341.33	350.20	310	6.31	7.27	350.20
320	256.28	285.78	310.38	327.42	339.38	348.60	356.51	320	7.81	8.19	356.51
330	286.81	311.13	327.93	339.76	348.91	356.79	4.32	330	11.32	10.41	4.32
340	318.23	332.82	343.43	351.97	359.62	7.23	15.64	340	21.60	16.75	15.64
350	341.79	350.59	358.33	5.89	14.09	23.98	37.24	350	52.76	37.83	37.24
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360			90.00

$e = 0.50 \quad i = 75^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	3.97	8.50	14.51	24.15	36.92	78.56	0°	90.00
10	9.66	16.22	27.27	51.43	102.71	86.05	158.52	10	158.52
20	24.81	45.56	92.93	137.48	156.50	23.83	171.76	20	171.76
30	59.73	113.46	146.57	160.66	168.33	7.89	177.76	30	177.76
40	115.25	147.30	161.02	168.55	173.72	4.27	181.81	40	181.81
50	143.15	159.04	167.34	172.82	177.13	2.89	185.11	50	185.11
60	154.81	164.91	171.10	175.71	179.69	2.12	188.10	60	188.10
70	161.03	168.56	173.72	177.91	181.81	1.88	191.03	70	191.03
80	164.99	171.16	175.75	179.74	183.69	1.60	194.03	80	194.03
90	167.82	173.17	177.44	181.34	185.43	1.47	197.25	90	197.25
100	170.00	174.83	178.90	182.81	187.11	1.38	200.81	100	200.81
110	171.78	176.26	180.21	184.19	188.78	1.35	204.87	110	204.87
120	173.29	177.53	181.44	185.54	190.47	1.34	209.64	120	209.64
130	174.62	178.70	182.61	186.88	192.25	1.34	215.40	130	215.40
140	175.82	179.80	183.75	188.24	194.14	1.36	222.49	140	222.49
150	176.93	180.85	184.89	189.64	196.19	1.40	231.31	150	231.31
160	177.99	181.89	186.05	191.14	198.49	1.50	242.29	160	242.29
170	179.00	182.92	187.24	192.74	201.09	1.60	255.39	170	255.39
180	180.00	183.97	188.50	194.51	204.15	1.77	270.00	180	270.00
190	181.00	185.05	189.85	196.50	207.81	1.99	284.61	190	284.61
200	182.01	186.19	191.33	198.79	212.31	2.29	297.71	200	297.71
210	183.07	187.42	192.99	201.51	218.06	2.72	308.69	210	308.69
220	184.18	188.76	194.89	204.83	225.61	3.32	317.51	220	317.51
230	185.38	190.28	197.15	209.04	235.76	4.21	324.60	230	324.60
240	186.71	192.03	199.92	214.64	249.46	5.60	330.36	240	330.36
250	188.22	194.12	203.45	222.41	266.83	7.77	335.13	250	335.13
260	190.00	196.72	208.23	233.76	286.06	11.35	339.19	260	339.19
270	192.18	200.16	215.15	250.70	303.77	16.94	342.75	270	342.75
280	195.01	205.04	226.09	273.90	317.95	23.20	345.97	280	345.97
290	198.97	212.67	244.68	298.56	328.65	24.66	348.97	290	348.97
300	205.19	226.46	274.55	318.27	336.84	19.71	351.90	300	351.90
310	216.85	254.70	306.78	331.99	343.39	13.72	354.89	310	354.89
320	244.75	298.62	328.68	341.70	348.98	9.71	358.19	320	358.19
330	300.27	329.34	342.03	349.19	354.21	7.49	2.24	330	2.24
340	335.19	345.12	351.24	355.82	359.80	6.63	6.00	340	6.00
350	350.34	355.10	359.13	3.06	7.40	7.24	13.24	350	13.24
360	0.00	3.97	8.50	14.51	24.15	11.45	68.52	360	68.52

$e = 0.60 \quad i = 15^\circ$

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	14.51	45.63	29.15	46.37	44.01	59.13	90.00
10	45.00	60.14	32.53	75.52	32.64	91.02	106.52	136.98
20	45.00	92.67	20.89	108.16	20.65	123.49	138.57	167.99
30	98.11	113.56	14.16	128.81	13.91	143.79	158.53	187.58
40	112.45	127.72	10.36	142.72	10.18	157.47	172.04	201.08
50	122.99	138.08	8.06	152.90	7.94	167.52	182.02	211.20
60	131.20	146.14	6.61	160.84	6.52	175.38	189.88	219.24
70	137.92	152.75	5.60	167.36	5.55	181.86	196.38	225.94
80	143.61	158.35	4.90	172.91	4.86	187.40	201.96	231.70
90	148.58	163.25	4.37	177.77	4.36	192.27	206.88	236.79
100	153.01	167.62	3.99	182.13	3.98	196.65	211.31	241.37
110	157.04	171.61	3.70	186.11	3.70	200.66	215.38	245.58
120	160.78	175.31	3.49	189.81	3.50	204.39	219.17	249.51
130	164.29	178.80	3.32	193.31	3.33	207.93	222.77	253.22
140	167.61	182.12	3.19	196.64	3.22	211.30	226.20	256.75
150	170.82	185.31	3.11	199.86	3.13	214.57	229.52	260.17
160	173.92	188.42	3.06	202.99	3.09	217.74	232.76	263.49
170	176.98	191.48	3.03	206.08	3.07	220.88	235.95	266.76
180	180.00	194.51	3.04	209.15	3.08	224.01	239.13	270.00
190	183.02	197.55	3.08	212.23	3.12	227.14	242.32	273.24
200	186.08	200.63	3.13	215.35	3.18	230.32	245.55	276.51
210	189.18	203.76	3.24	218.53	3.29	233.56	248.84	279.83
220	192.39	207.00	3.37	221.82	3.43	236.91	252.24	283.25
230	195.71	210.37	3.55	225.25	3.62	240.39	255.77	286.78
240	199.22	213.92	3.79	228.87	3.86	244.08	259.50	290.49
250	202.96	217.71	4.10	232.73	4.17	248.00	263.46	294.42
260	206.99	221.81	4.51	236.90	4.58	252.23	267.72	298.63
270	211.42	226.32	5.06	241.48	5.15	256.88	272.39	303.21
280	216.39	231.38	5.79	246.63	5.88	262.07	277.60	308.30
290	222.08	237.17	6.84	252.51	6.92	268.00	283.51	314.06
300	228.80	244.01	8.33	259.43	8.41	274.95	290.42	320.76
310	237.01	252.34	10.66	267.84	10.69	283.35	298.74	328.80
320	247.55	263.00	14.41	278.53	14.33	293.97	309.21	338.92
330	261.89	277.41	20.83	292.86	20.55	308.12	323.12	352.42
340	282.84	298.24	31.64	313.41	31.12	328.31	342.98	367.51
350	315.00	329.88	44.63	344.53	44.62	359.04	449.7	430.2
360	0.00	14.51		29.15		44.01	59.13	90.00

$e = 0.60 \quad i = 30^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	13.06	26.57	40.89	56.31	72.81	90.00	0°	50.08
10	41.88	44.31	47.36	50.25	52.00	51.93	140.08	10	29.12
20	33.85	35.61	36.16	35.28	33.33	31.07	169.20	20	17.60
30	23.30	22.96	21.80	20.30	18.93	17.98	186.80	30	
40	99.03	14.84	13.80	12.88	12.28	12.07	199.67	40	12.27
50	114.75	10.38	9.67	9.17	8.96	8.95	208.50	50	9.43
60	125.91	7.81	7.34	7.09	7.06	7.28	216.22	60	7.72
70	134.32	6.69	5.93	5.81	5.90	6.16	222.81	70	6.59
80	141.01	5.54	5.00	4.97	5.11	5.41	228.62	80	5.81
90	146.55	4.74	4.37	4.40	4.56	4.86	233.86	90	5.24
100	151.29	4.17	3.91	3.97	4.17	4.46	238.67	100	4.81
110	155.46	3.74	3.57	3.67	3.87	4.17	243.14	110	4.47
120	159.20	3.44	3.33	3.45	3.67	3.95	247.37	120	4.23
130	162.64	3.20	3.16	3.29	3.52	3.79	251.41	130	4.04
140	165.84	3.02	3.03	3.18	3.40	3.67	255.29	140	3.88
150	168.86	2.89	2.94	3.11	3.34	3.59	259.06	150	3.77
160	171.75	2.80	2.88	3.05	3.30	3.52	262.75	160	3.69
170	174.55	2.74	2.87	3.06	3.29	3.52	266.39	170	3.64
180	177.29	2.71	2.88	3.07	3.31	3.52	270.00	180	3.61
190	180.00	2.71	2.91	3.13	3.36	3.55	273.61	190	3.61
200	182.71	2.74	2.97	3.20	3.44	3.61	277.25	200	3.64
210	185.45	2.80	3.07	3.30	3.54	3.69	280.94	210	3.69
220	188.25	2.89	3.22	3.47	3.69	3.81	284.71	220	3.77
230	191.14	3.02	3.38	3.65	3.87	3.96	288.59	230	3.88
240	194.16	3.20	3.63	3.90	4.11	4.17	292.63	240	4.04
250	197.36	3.44	3.92	4.20	4.39	4.40	296.86	250	4.23
260	200.80	3.74	4.31	4.59	4.75	4.70	301.33	260	4.47
270	204.54	4.17	4.80	5.10	5.21	5.09	306.14	270	4.81
280	208.71	4.74	5.48	5.74	5.79	5.60	311.38	280	5.24
290	213.45	5.54	6.37	6.60	6.55	6.23	317.19	290	5.81
300	218.99	6.69	7.61	7.75	7.54	7.11	323.78	300	6.59
310	225.68	8.41	9.35	9.30	8.90	8.30	331.50	310	7.72
320	234.09	11.16	11.90	11.55	10.85	10.07	340.93	320	9.43
330	245.25	15.72	15.70	14.82	13.77	12.86	353.20	330	12.27
340	260.97	23.30	23.35	19.85	18.59	17.81	372.60	340	17.60
350	284.27	33.85	31.60	28.10	27.53	27.88	408.20	350	29.12
360	318.12	41.88	40.51	41.75	44.12	47.16	500.00	360	50.08
			26.57	40.89	56.31	72.81	90.00		

		$e = 0.60 \quad i = 45^\circ$										M
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	$\omega = 90^\circ$	$\omega = 90^\circ$	$\omega = 90^\circ$	M	
0°	0.00	10.73	41.17	22.21	48.35	35.26	56.14	50.77	61.29	69.25	0°	
10	36.20	51.90	70.56	70.56	48.35	91.40	56.14	112.06	61.29	130.34	10	
20	72.70	93.65	114.14	114.14	43.58	132.10	40.70	147.13	35.07	159.85	20	
30	101.02	120.78	137.69	137.69	23.55	151.81	19.71	163.93	16.80	174.92	30	
40	18.42	136.57	150.87	150.87	13.18	163.11	11.30	174.15	10.22	184.79	40	
50	131.57	146.68	159.47	159.47	8.60	170.79	7.68	181.48	7.33	192.27	50	
60	140.10	153.84	165.73	165.73	6.26	176.62	5.83	187.27	5.79	198.42	60	
70	146.53	159.34	170.68	170.68	4.95	181.36	4.74	192.15	4.88	203.78	70	
80	151.66	163.80	174.80	174.80	4.12	185.43	4.07	196.45	4.30	208.62	80	
90	155.91	167.57	178.37	178.37	3.57	189.05	3.62	200.36	3.91	213.12	90	
100	159.55	170.87	181.56	181.56	3.19	192.35	3.30	204.00	3.64	217.37	100	
110	162.77	173.84	184.48	184.48	2.92	195.43	3.08	207.47	3.47	221.45	110	
120	165.68	176.57	187.21	187.21	2.73	198.37	2.94	210.82	3.35	225.44	120	
130	168.36	179.12	189.82	189.82	2.61	201.21	2.84	214.10	3.28	229.38	130	
140	170.87	181.55	192.34	192.34	2.52	203.99	2.78	217.36	3.26	233.28	140	
150	173.25	183.90	194.81	194.81	2.47	206.77	2.77	220.62	3.30	237.20	150	
160	175.54	186.18	197.25	197.25	2.44	209.54	2.83	223.92	3.37	241.14	160	
170	177.79	188.45	199.71	199.71	2.46	212.37	2.89	227.29	3.48	245.15	170	
180	180.00	190.73	202.21	202.21	2.50	215.26	3.01	230.77	3.61	249.25	180	
190	182.21	193.04	204.77	204.77	2.56	218.27	3.15	234.38	3.78	253.45	190	
200	184.46	195.41	207.44	207.44	2.67	221.42	3.33	238.16	3.97	257.78	200	
210	186.75	197.86	210.24	210.24	2.80	224.75	3.57	242.13	4.24	262.25	210	
220	189.13	200.45	213.23	213.23	2.99	228.32	3.86	246.37	4.53	266.90	220	
230	191.64	203.22	216.44	216.44	3.53	232.18	4.24	250.90	4.89	271.75	230	
240	194.32	206.21	219.97	219.97	3.92	236.42	4.68	255.79	5.31	276.84	240	
250	197.23	209.51	223.89	223.89	4.42	241.10	5.26	261.10	5.79	282.18	250	
260	200.45	213.22	228.31	228.31	5.11	246.36	5.98	266.89	6.38	287.81	260	
270	204.09	217.47	233.42	233.42	6.02	252.34	6.89	273.27	7.05	293.79	270	
280	208.34	222.50	239.44	239.44	7.27	259.23	8.04	280.32	7.86	300.16	280	
290	213.47	228.61	246.71	246.71	8.99	267.27	9.47	288.18	8.78	307.02	290	
300	219.90	236.33	255.70	255.70	11.35	276.74	11.22	296.96	9.88	314.48	300	
310	228.43	246.50	267.05	267.05	14.52	287.96	13.31	306.84	11.26	322.74	310	
320	240.56	260.48	281.57	281.57	18.37	301.27	15.72	318.10	13.12	332.16	320	
330	258.98	280.07	299.94	299.94	22.33	316.99	18.69	331.22	16.15	343.42	330	
340	287.30	306.27	322.27	322.27	26.28	335.68	23.62	347.37	22.64	358.18	340	
350	323.80	336.99	348.55	348.55	33.66	359.30	35.96	10.01	40.76	21.41	350	
360	0.00	10.73	22.21	22.21	35.26	50.77	50.77	69.25	90.00	90.00	360	

$e = 0.60 \quad i = 60^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	7.63	34.41	65.41	78.93	81.81	90.00	0°	
10	27.37	42.04	53.11	65.41	78.93	81.81	154.22	10	64.22
20	66.23	95.15	122.36	141.96	155.44	165.46	173.72	20	19.50
30	105.40	130.11	147.23	159.25	168.49	176.41	183.94	30	10.22
40	128.60	146.20	158.49	167.88	175.86	183.39	191.29	40	7.35
50	141.43	155.07	165.17	173.46	181.05	188.74	197.41	50	6.12
60	149.41	160.85	169.81	177.61	185.15	193.25	202.92	60	5.51
70	154.95	165.07	173.38	180.97	188.65	197.31	208.14	70	5.22
80	159.12	168.39	176.31	183.85	191.79	201.10	213.24	80	5.10
90	162.45	171.14	178.84	186.42	194.70	204.76	218.33	90	5.09
100	165.23	173.52	181.10	188.80	197.48	208.37	223.48	100	5.15
110	167.63	175.64	183.17	191.04	200.18	211.99	228.75	110	5.27
120	169.77	177.57	185.12	193.21	202.87	215.68	234.17	120	5.42
130	171.71	179.38	186.98	195.35	205.59	219.50	239.78	130	5.61
140	173.51	181.10	188.79	197.47	208.36	223.47	245.55	140	5.77
150	175.22	182.76	190.59	199.63	211.24	227.65	251.50	150	5.95
160	176.85	184.38	192.39	201.84	214.25	232.07	257.57	160	6.07
170	178.43	186.00	194.22	204.14	217.46	236.78	263.76	170	6.19
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180	6.24
190	181.57	189.30	198.07	209.15	224.62	247.20	276.24	190	6.24
200	183.15	191.03	200.16	211.96	228.71	252.96	282.43	200	6.19
210	184.78	192.84	202.40	215.03	233.21	259.10	288.50	210	6.07
220	186.49	194.77	204.85	218.46	238.25	265.63	294.45	220	5.95
230	188.29	196.87	207.57	222.33	243.91	272.48	300.22	230	5.77
240	190.23	199.20	210.66	226.80	250.30	279.63	305.83	240	5.61
250	192.37	201.82	214.22	232.03	257.51	286.98	311.25	250	5.42
260	194.77	204.84	218.45	238.24	265.61	294.44	316.52	260	5.27
270	197.55	208.46	223.61	245.76	274.61	301.94	321.67	270	5.15
280	200.88	212.94	230.14	254.94	284.45	309.42	326.76	280	5.09
290	205.05	218.74	238.67	266.14	294.91	316.84	331.86	290	5.42
300	210.59	226.71	250.17	279.49	305.73	324.25	337.08	300	5.51
310	218.57	238.41	265.83	294.63	316.65	331.73	342.59	310	5.51
320	231.40	256.66	286.15	310.66	327.61	339.52	348.71	320	6.12
330	254.60	284.10	309.16	326.59	338.77	348.11	356.06	330	7.35
340	293.77	316.06	331.31	342.28	351.00	358.71	6.28	340	10.22
350	332.63	343.28	351.85	359.51	7.11	15.50	25.78	350	19.50
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360	64.22

$e = 0.60 \quad i = 75^\circ$									
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	
0°	0.00	3.97	8.50	14.51	24.15	44.01	90.00	0°	
10	15.00	21.05	37.55	93.82	137.91	156.69	165.96	10	75.96
20	34.61	74.86	94.71	157.95	166.69	172.35	176.74	20	10.78
	49.61	48.55	20.81	10.95	7.29	5.79	5.30		
30	118.01	148.43	161.57	168.90	173.98	178.14	182.04	30	3.86
40	147.04	12.46	168.47	173.66	177.85	181.76	185.90	40	3.32
50	157.57	5.58	172.19	176.61	180.54	184.55	189.22	50	3.13
	5.41	3.34	2.49	2.15	2.13	2.40			
60	162.98	169.81	174.68	178.76	182.67	186.95	192.85	60	
70	166.40	2.33	176.56	180.50	184.50	189.16	195.48	70	3.13
80	168.83	1.79	178.09	181.99	186.17	191.30	198.74	80	3.26
	1.87	1.46	1.31	1.35	1.56	2.13	3.52		
90	170.70	175.39	179.40	183.34	187.73	193.43	202.26	90	3.89
100	172.23	1.53	180.57	184.58	189.26	195.62	206.15	100	4.40
110	173.52	1.29	181.64	185.77	190.77	197.92	210.55	110	5.09
	1.14	1.00	1.01	1.16	1.55	2.47			
120	174.66	178.74	182.65	186.93	192.32	200.39	215.64	120	5.98
130	175.69	0.94	183.63	188.09	193.92	203.11	221.62	130	7.08
140	176.63	0.89	184.58	189.25	195.61	206.14	228.70	140	8.41
	0.89	0.86	0.95	1.21	1.82	3.45			
150	177.52	181.43	185.53	190.46	197.43	209.59	237.11	150	9.82
160	178.37	0.84	186.48	191.72	199.42	213.59	246.93	160	11.14
170	179.19	0.86	187.47	193.06	201.63	218.32	258.07	170	11.93
	0.81	0.86	1.03	1.45	2.52	5.69			
180	180.00	183.97	188.50	194.51	204.15	224.01	270.00	180	
190	180.81	0.87	189.59	196.11	207.06	230.92	281.93	190	11.93
200	181.63	0.92	190.76	197.90	210.51	239.38	293.07	200	11.14
	0.85	0.97	1.28	2.04	4.18	10.21	9.82		
210	182.48	186.73	192.04	199.94	214.69	249.59	302.89	210	
220	183.37	1.04	193.48	202.35	219.92	261.60	311.30	220	8.41
230	184.31	1.15	195.12	205.25	226.58	274.78	318.38	230	7.08
	1.03	1.30	1.94	3.62	8.75	13.37	5.98		
240	185.34	190.22	197.06	208.87	235.33	288.15	324.36	240	
250	186.48	1.49	199.40	213.55	246.83	300.53	329.45	250	5.09
260	187.77	1.77	202.34	219.90	261.56	311.28	333.85	260	4.40
	1.53	2.19	3.91	9.08	17.30	9.01	3.89		
270	189.30	195.67	206.25	228.98	278.86	320.29	337.74	270	
280	191.17	2.87	211.80	242.54	296.46	327.80	341.26	280	3.52
290	193.60	4.01	220.37	262.58	311.89	334.11	344.52	290	3.26
	3.42	6.24	14.77	25.32	12.37	5.45	3.13		
300	197.02	208.79	235.14	287.90	324.26	339.56	347.65	300	
310	202.43	5.41	268.83	311.53	333.96	344.44	350.78	310	3.13
320	212.96	10.53	37.26	328.92	341.82	349.06	354.10	320	3.32
	29.03	50.50	28.33	12.23	6.81	4.72	3.86		
330	241.99	295.89	327.56	341.15	348.63	353.78	357.96	330	
340	310.39	333.49	344.19	350.61	355.31	359.33	326	5.30	
350	345.00	351.16	355.76	359.74	3.69	8.17	14.04	350	10.78
	15.00	12.81	12.74	14.77	20.46	35.84	35.84		75.96
360	0.00	3.97	8.50	14.51	24.15	44.01	90.00		

$e = 0.70 \quad i = 15^\circ$

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	14.51	29.15	44.01	59.13	74.50	90.00	0°
10	64.55	65.47	66.35	66.96	67.12	66.78	66.05	10
20	33.36	33.37	33.10	32.62	32.08	31.61	31.33	20
	97.91	113.35	128.60	143.59	158.33	172.89	187.38	30
30	115.85	131.06	146.01	160.71	175.25	189.74	204.33	40
40	127.29	142.30	157.06	171.63	186.13	200.68	215.40	50
50	135.49	150.36	165.00	179.52	194.03	208.66	223.51	60
60	141.85	156.61	171.19	185.68	200.23	214.94	229.90	70
70	147.03	161.72	176.25	190.75	205.34	220.14	235.19	80
80	151.44	166.07	180.58	195.09	209.74	224.61	239.74	90
90	155.30	169.89	184.39	198.92	213.62	228.56	243.76	100
100	158.76	173.31	187.81	202.37	217.12	232.12	247.38	110
110	161.91	176.44	190.94	205.54	220.34	235.40	250.71	120
120	164.84	179.35	193.86	208.49	223.34	238.45	253.81	130
130	167.59	182.10	196.62	211.28	226.18	241.34	256.73	140
140	170.22	184.71	199.25	213.95	228.90	244.11	259.53	150
150	172.74	187.24	201.80	216.54	231.53	246.78	262.23	160
160	175.20	189.70	204.28	219.05	234.09	249.38	264.86	170
170	177.61	192.12	206.72	221.54	236.62	251.95	267.44	180
180	180.00	194.51	209.15	224.01	239.13	254.50	270.00	190
190	182.39	196.91	211.58	226.48	241.65	257.04	272.56	200
200	184.80	199.34	214.04	228.99	244.20	259.62	275.14	210
210	187.26	201.82	216.56	231.55	246.80	262.25	277.77	220
220	189.78	204.37	219.14	234.18	249.48	264.95	280.47	230
230	192.41	207.02	221.84	236.93	252.26	267.75	283.27	240
240	195.16	209.81	224.68	239.81	255.19	270.69	286.19	250
250	198.09	212.77	227.69	242.88	258.29	273.81	289.29	260
260	201.24	215.97	230.95	246.20	261.64	277.16	292.62	270
270	204.70	219.48	234.53	249.83	265.30	280.82	296.24	280
280	208.56	223.41	238.52	253.88	269.38	284.89	300.26	290
290	212.97	227.89	243.09	258.50	274.02	289.50	304.81	300
300	218.15	233.17	248.45	263.91	279.44	294.87	310.10	310
310	224.51	239.64	255.01	270.52	286.02	301.38	316.49	320
320	232.71	247.98	263.44	278.96	294.40	309.63	324.60	330
330	244.15	259.57	275.09	290.56	305.86	320.90	335.67	340
340	262.09	277.62	293.07	308.32	323.32	338.06	352.62	350
350	295.45	310.67	325.62	340.33	354.87	9.37	23.95	360
	64.55	63.84	63.53	63.68	64.26	74.50	66.05	
		14.51	29.15	44.01	59.13		90.00	

$e = 0.70 \quad i = 30^\circ$

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	13.06	26.57	40.89	56.31	72.81	90.00	0°
10	62.04	65.79	69.56	72.25	72.97	71.48	158.29	10
20	98.80	115.72	131.69	146.53	160.39	173.62	186.63	20
30	118.39	134.18	148.84	162.58	175.74	188.75	202.06	30
40	130.35	145.28	159.22	172.49	185.50	198.69	212.50	40
50	138.61	152.97	166.50	179.57	192.62	206.10	220.40	50
60	144.84	158.81	172.09	185.10	198.28	212.06	226.80	60
70	149.82	163.50	176.64	189.66	203.01	217.09	232.21	70
80	153.99	167.46	180.52	193.59	207.12	221.49	236.95	80
90	157.59	170.92	183.93	197.08	210.80	225.44	241.20	90
100	160.79	174.00	187.01	200.26	214.16	229.06	245.08	100
110	163.68	176.81	189.84	203.19	217.28	232.42	248.67	110
120	166.35	179.42	192.47	205.95	220.23	235.60	252.05	120
130	168.84	181.88	194.98	208.58	223.05	238.63	255.27	130
140	171.21	184.23	197.39	211.12	225.78	241.57	258.35	140
150	173.49	186.50	199.73	213.60	228.45	244.43	261.34	150
160	175.69	188.71	202.02	216.03	231.07	247.24	264.27	160
170	177.86	190.89	204.29	218.46	233.69	250.03	267.15	170
180	180.00	193.06	206.57	220.89	236.31	252.81	270.00	180
190	182.14	195.25	208.86	223.35	238.96	255.61	272.85	190
200	184.31	197.47	211.20	225.87	241.67	258.46	275.73	200
210	186.51	199.74	213.61	228.47	244.45	261.37	278.66	210
220	188.79	202.10	216.12	231.17	247.34	264.37	281.65	220
230	191.16	204.57	218.76	234.01	250.36	267.49	284.73	230
240	193.65	207.18	221.56	237.03	253.57	270.77	287.95	240
250	196.32	209.99	224.57	240.27	256.98	274.25	291.33	250
260	199.21	213.05	227.86	243.80	260.69	277.98	294.92	260
270	202.41	216.45	231.52	247.72	264.76	282.03	298.80	270
280	206.01	220.30	235.67	252.13	269.31	286.51	303.05	280
290	210.18	224.77	240.48	257.21	274.48	291.55	307.79	290
300	215.16	230.13	246.23	263.22	280.50	297.34	313.20	300
310	221.39	236.84	253.37	270.58	287.76	304.22	319.60	310
320	229.65	245.72	262.69	279.97	296.84	312.73	327.50	320
330	241.61	258.40	275.68	292.71	308.87	323.92	337.94	330
340	261.20	278.48	295.41	311.40	326.26	340.14	353.37	340
350	297.96	313.78	328.47	342.23	355.40	8.41	21.71	350
360	0.00	13.06	26.57	40.89	56.31	72.81	90.00	360

		$e = 0.70 \quad i = 45^\circ$											
M	M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$				M	M
0°	0°	0.00	10.73	65.69	22.21	75.28	35.26	82.37	50.77	84.28	69.25	90.00	0°
10	10	56.97	76.42	44.11	97.49	39.99	117.63	82.37	135.05	28.73	149.59	161.99	10
20	20	100.74	120.53	19.43	137.48	16.25	151.64	13.99	163.78	12.74	174.78	185.42	20
30	30	123.50	139.96	10.54	153.73	9.06	165.63	8.22	176.52	7.97	187.16	198.31	30
40	40	136.13	150.50	6.89	162.79	6.12	173.85	5.80	184.49	5.87	195.44	207.49	40
50	50	144.26	157.39	5.04	168.91	4.61	179.65	4.52	190.36	4.74	201.80	214.79	50
60	60	150.10	162.43	3.98	173.52	3.73	184.17	3.74	195.10	4.02	207.09	221.01	60
70	70	154.60	166.41	3.30	177.25	3.17	187.91	3.26	199.12	3.57	211.69	226.48	70
80	80	158.28	169.71	2.85	180.42	2.79	191.17	2.92	202.69	3.26	215.83	231.45	80
90	90	161.39	172.56	2.53	183.21	2.52	194.09	2.68	205.95	3.04	219.66	236.05	90
100	100	164.12	175.09	2.31	185.73	2.33	196.77	2.51	208.99	2.88	223.26	240.36	100
110	110	166.55	177.40	2.13	188.06	2.18	199.28	2.39	211.87	2.77	226.70	244.45	110
120	120	168.78	179.53	2.00	190.24	2.08	201.67	2.31	214.64	2.70	230.01	248.36	120
130	130	170.85	181.53	1.92	192.32	2.02	203.98	2.26	217.34	2.66	233.26	252.15	130
140	140	172.81	183.45	1.86	194.34	1.98	206.24	2.23	220.00	2.65	236.45	255.83	140
150	150	174.67	185.31	1.82	196.32	1.95	208.47	2.24	222.65	2.66	239.63	259.44	150
160	160	176.48	187.13	1.80	198.27	1.96	210.71	2.26	225.31	2.71	242.80	262.99	160
170	170	178.25	188.93	1.80	200.23	1.98	212.97	2.29	228.02	2.75	246.01	266.51	170
180	180	180.00	190.73	1.82	202.21	2.02	215.26	2.37	230.77	2.83	249.25	270.00	180
190	190	181.75	192.55	1.86	204.23	2.09	217.63	2.46	233.60	2.96	252.55	273.49	190
200	200	183.52	194.41	1.92	206.32	2.17	220.09	2.58	236.56	3.09	255.96	277.01	200
210	210	185.33	196.33	2.01	208.49	2.30	222.67	2.74	239.65	3.27	259.47	280.56	210
220	220	187.19	198.34	2.13	210.79	2.45	225.41	2.94	242.92	3.48	263.12	284.17	220
230	230	189.15	200.47	2.28	213.24	2.66	228.35	3.18	246.40	3.74	266.93	287.85	230
240	240	191.22	202.75	2.48	215.90	2.91	231.53	3.49	250.14	4.05	270.95	291.64	240
250	250	193.45	205.23	2.75	218.81	3.25	235.02	3.91	254.19	4.46	275.20	295.55	250
260	260	195.88	207.98	3.11	222.06	3.71	238.93	4.42	258.65	4.94	279.74	299.64	260
270	270	198.61	211.09	3.61	225.77	4.32	243.35	5.10	263.59	5.56	284.63	303.95	270
280	280	201.72	214.70	4.31	230.09	5.17	248.45	6.02	269.15	6.33	289.96	308.55	280
290	290	205.40	219.01	5.34	235.26	6.39	254.47	7.24	275.48	7.31	295.81	313.52	290
300	300	209.90	224.35	6.99	241.65	8.26	261.71	9.00	282.79	8.62	302.34	318.99	300
310	310	215.74	231.34	9.74	249.91	11.16	270.71	11.45	291.41	10.37	309.80	325.21	310
320	320	223.87	241.08	14.81	261.07	15.88	282.16	14.97	301.78	12.85	318.53	332.52	320
330	330	236.50	255.89	24.46	276.95	23.24	297.13	20.06	314.63	16.76	329.25	341.69	330
340	340	259.26	280.35	39.22	300.19	33.21	317.19	28.14	331.39	24.85	343.57	354.58	340
350	350	303.03	319.57	51.16	333.40	48.81	345.33	49.93	356.24	54.53	6.88	371.99	350
360	360	0.00	10.73	22.21	35.26	50.77	69.25	82.37	90.00	90.00	90.00	90.00	360

$e = 0.70 \quad i = 60^\circ$

M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M
0°	0.00	7.63	16.10	26.57	40.89	61.81	90.00	0°
10	47.40	63.51	100.53	126.51	144.78	157.46	167.05	10
20	105.02	129.83	147.04	159.11	168.38	176.30	183.84	20
30	133.11	149.28	160.76	169.73	177.54	185.08	193.17	30
40	145.80	158.20	167.64	175.64	183.18	191.05	200.19	40
50	153.03	163.59	172.11	179.75	187.37	195.80	206.17	50
60	157.87	167.38	175.41	182.95	190.80	199.88	211.58	60
70	161.44	170.30	178.06	185.61	193.78	203.58	216.67	70
80	164.27	172.68	180.30	187.95	196.47	207.05	221.58	80
90	166.61	174.73	182.27	190.06	198.99	210.38	226.40	90
100	168.62	176.53	184.06	192.03	201.39	213.64	231.17	100
110	170.41	178.16	185.72	193.89	203.73	216.88	235.93	110
120	172.02	179.67	187.28	195.69	206.03	220.13	240.71	120
130	173.50	181.09	188.78	197.46	208.34	223.44	245.51	130
140	174.90	182.44	190.25	199.22	210.68	226.84	250.35	140
150	176.23	183.76	191.70	200.98	213.08	230.35	255.23	150
160	177.51	185.05	193.14	202.78	215.56	233.99	260.13	160
170	178.76	186.34	194.61	204.64	218.16	237.81	265.07	170
180	180.00	187.63	196.10	206.57	220.89	241.81	270.00	180
190	181.24	188.94	197.65	208.59	223.81	246.04	274.93	190
200	182.49	190.30	199.28	210.76	226.96	250.52	279.87	200
210	183.77	191.71	201.00	213.10	230.37	255.27	284.77	210
220	185.10	193.19	202.85	215.65	234.13	260.31	289.65	220
230	186.50	194.79	204.87	218.48	238.29	265.67	294.49	230
240	187.98	196.52	207.10	221.67	242.94	271.34	299.29	240
250	189.59	198.43	209.63	225.31	248.18	277.33	304.07	250
260	191.38	200.59	212.54	229.56	254.15	283.64	308.83	260
270	193.39	203.10	216.00	234.63	260.98	290.27	313.60	270
280	195.73	206.09	220.21	240.82	268.80	297.18	318.42	280
290	198.56	209.80	225.56	248.54	277.72	304.37	323.33	290
300	202.13	214.65	232.66	258.36	287.80	311.85	328.42	300
310	206.97	221.47	242.65	271.00	299.01	319.67	333.83	310
320	214.20	232.00	257.47	286.94	311.23	327.99	339.81	320
330	226.89	250.43	279.77	305.93	324.39	337.18	346.83	330
340	254.98	284.48	309.44	326.78	338.91	348.22	356.16	340
350	312.60	57.62	340.50	349.52	357.34	4.88	12.95	350
360	0.00	7.63	16.10	26.57	40.89	61.81	90.00	360

$e = 0.70 \quad i = 75^\circ$											
M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$	$\omega = 45^\circ$	$\omega = 60^\circ$	$\omega = 75^\circ$	$\omega = 90^\circ$	M	$\omega = 0^\circ$	$\omega = 15^\circ$	$\omega = 30^\circ$
0°	0.00	29.38	52.62	8.50	101.25	14.51	130.52	24.15	135.78	44.01	123.87
10	29.38	88.02	56.59	109.75	51.70	145.03	23.79	159.93	13.99	167.88	10.21
20	117.40	33.66	148.18	161.45	8.31	168.82	5.82	173.92	4.80	178.09	4.54
30	151.06	9.56	162.90	169.76	3.77	174.64	3.10	178.72	2.93	182.63	3.14
40	160.62	4.62	168.30	173.53	2.36	177.74	2.13	181.65	2.18	185.77	2.56
50	165.24	2.87	171.33	175.89	1.73	179.87	1.66	183.83	1.81	188.33	2.27
60	168.11	2.03	173.39	177.62	1.37	181.53	1.38	185.64	1.59	190.60	2.13
70	170.14	1.56	174.94	178.99	1.17	182.91	1.22	187.23	1.47	192.73	2.07
80	171.70	1.28	176.20	180.16	1.02	184.13	1.12	188.70	1.40	194.80	2.08
90	172.98	1.07	177.26	181.18	0.92	185.25	1.04	190.10	1.36	196.88	2.13
100	174.05	0.95	178.20	182.10	0.87	186.29	1.01	191.46	1.36	199.01	2.21
110	175.00	0.85	179.05	182.97	0.81	187.30	0.97	192.82	1.37	201.22	2.36
120	175.85	0.78	179.83	183.78	0.79	188.27	0.98	194.19	1.41	203.58	2.54
130	176.63	0.72	180.56	184.57	0.78	189.25	0.98	195.60	1.47	206.12	2.78
140	177.35	0.70	181.27	185.35	0.77	190.23	1.00	197.07	1.56	208.90	3.09
150	178.05	0.66	181.95	186.12	0.77	191.23	1.04	198.63	1.68	211.99	3.47
160	178.71	0.65	182.62	186.89	0.79	192.27	1.09	200.31	1.82	215.46	3.97
170	179.36	0.64	183.29	187.68	0.82	193.36	1.15	202.13	2.02	219.43	4.58
180	180.00	0.64	183.97	188.50	0.85	194.51	1.25	204.15	2.26	224.01	5.34
190	180.64	0.65	184.66	189.35	0.91	195.76	1.37	206.41	2.59	229.35	6.30
200	181.29	0.66	185.37	190.26	0.98	197.13	1.52	209.00	3.01	235.65	7.42
210	181.95	0.70	186.12	191.24	1.06	198.65	1.72	212.01	3.58	243.07	8.67
220	182.65	0.72	186.92	192.30	1.19	200.37	2.00	215.59	4.36	251.74	9.93
230	183.37	0.78	187.78	193.49	1.35	202.37	2.36	219.95	5.42	261.67	10.92
240	184.15	0.85	188.73	194.84	1.56	204.73	2.89	225.37	6.91	272.59	12.37
250	185.00	0.95	189.79	196.40	1.88	207.62	3.65	232.28	8.97	283.96	14.16
260	185.95	1.07	191.00	198.28	2.83	211.27	4.83	241.25	11.69	295.12	16.38
270	187.02	1.28	192.45	200.61	3.03	216.10	6.73	252.94	14.74	305.50	18.37
280	188.30	1.56	194.22	203.64	4.19	222.83	9.96	267.68	17.00	314.77	20.37
290	189.86	2.03	196.51	207.83	6.33	232.79	15.51	284.68	17.13	322.88	22.37
300	191.89	2.87	199.69	214.16	10.86	248.30	23.63	301.81	15.17	332.97	24.37
310	194.76	4.62	204.58	225.02	21.75	271.93	28.55	316.98	12.45	336.28	26.37
320	199.38	9.56	213.52	248.77	41.63	300.48	23.99	329.43	10.23	342.07	28.37
330	208.94	33.66	235.52	288.40	89.42	324.47	16.80	339.66	9.05	347.71	30.37
340	242.60	88.02	296.52	327.82	21.79	341.27	13.26	348.71	9.91	353.84	32.37
350	330.62	29.38	342.68	349.61	18.89	354.53	19.98	358.62	25.53	358.01	34.37
360	0.00		3.97	8.50		14.51		24.15		44.01	

Summary

The first two chapters of this paper are concerned with the observational errors in micrometer measurements of position angles and angular distances in visual double stars. This investigation has been made with special regard to the use of these measurements for determining the orbits of visual binaries. — The last three chapters deal more directly with the computation of visual binary orbits.

Chapter I contains a survey of different methods for eliminating, or determining, the systematic errors in visual double star measurements. The method proposed by SALET and BOSLER for the direct elimination of the errors in position angle measurements is discussed. An account is also given of the various methods of determining the systematic errors of observation. A classification of such methods is given on p. 11. It may be remarked that such methods are no doubt worth a careful study, even though it is generally not advisable to use them for deriving corrections to the measurements.

In *Chapter II*, the residuals of the orbits of 42 binaries have been utilized for a study of the systematic and accidental errors in the measurements performed by O. STRUVE, W. DOBERCK, G. C. COMSTOCK, R. G. AITKEN, W. BRYANT, G. VAN BIESBROECK and W. H. VAN DEN BOS. The results are summarized on p. 61—65, where the different arguments to be considered are also discussed. In close pairs, like those occurring in this investigation, the angular distance between the stars is probably the most important argument for the systematic errors in the distance measurements. The smallest distances appear to be overestimated by most observers, while the somewhat larger distances are rather measured too small. (Cf. Fig. 14, p. 62.) The variations of the systematic errors with the time of observation usually seem to be of less importance, as far as changes of long duration are concerned. In general, it does not either seem probable that the inclination (with respect to the vertical) of the line joining the stars is of any great importance.

Chapter III gives an account of recent work on the computation of visual binary orbits. The papers reviewed here are mostly such as are not mentioned in the second edition of R. G. AITKEN's monograph *The Binary*

Stars, which was published in 1935. The different methods for orbit computation are mainly treated with regard to their practical applicability.

A new method for deriving preliminary orbits of visual binaries is developed in *Chapter IV*. In its present form the method is mainly based on the measurements of the position angles and should only be used for fairly open orbits. The principle of the method is the following: The position angles of the observations (or normal places) are plotted on transparent paper, against quantities which correspond to the mean anomalies. This diagram is compared with ephemeris curves which have been constructed in advance. For different values of eccentricity (e), inclination (i) and periastron longitude (ω), these ephemeris curves give the quantity $\theta_0 = \theta - \Omega$ (position angle of the companion *minus* position angle of the node) as a function of the mean anomaly. By means of the above comparison approximate values of five of the orbit elements are directly obtained, and then the determination of the orbit is easily completed. — The ephemeris curves are founded on tables which are given in the Appendix. — The use of the distance measurements, and a few other questions, are also discussed.

Chapter V contains the orbit determinations of the following ADS stars: 2612, 3098, 5234, 6762, 7334, 7871, 8419, 8635, 9397 and 12961. Five of these orbits are entirely new; for most of the others only unsatisfactory orbits were previously available. Six of the computed orbits have been determined by means of the ephemeris curve method given in Chapter IV. Some of the orbits have been subject to differential corrections according to one of the methods recently proposed by W. P. HIRST.

The *Appendix* contains the ephemeris tables. These give, for different values of e , i and ω , the quantity $\theta_0 = \theta - \Omega$ as a function of the mean anomaly M . Further explanations of the tables are given on p. 149.

Contents

	Page
Preface	5
Chapter I. Methods of eliminating or determining the systematic errors in visual double star measurements	7
Chapter II. Systematic and accidental errors in the measurements by different observers as obtained from computed orbits	28
Introduction	28
The material	28
General remarks	29
Otto Struve	33
William Doberck	42
George C. Comstock	43
Robert Grant Aitken	47
William Bryant	51
George van Biesbroeck	54
Willem H. van den Bos	58
Summary and discussion of the results	61
Chapter III. Recent work on the computation of visual binary star orbits	66
General remarks	66
Notations	67
Recent work on Thiele-Innes' method	68
Other analytical methods	72
The method of the hodographic ellipse	82
Some graphical methods	83
Danjon's method (and a few other methods)	85
Differential corrections	88
On the accuracy of orbit determinations	92
Parabolic orbits	94
Chapter IV. Ephemeris tables and ephemeris curves for the determination of the orbits of visual binaries	96
Remarks on the method	96
Computation of the tables	98
Construction of the curves	100
The orbit determination	102
Inflection points	107
Chapter V. The orbits of ten visual binaries	110
Introduction	110

	Page
List of observers	111
The orbit of ADS 2612 (AB) = BDS 1747 = Σ 400	112
The orbit of ADS 3098 = BDS 2088 = Σ 511	115
The orbit of ADS 5234 = BDS 3474 = O Σ 149	118
The orbit of ADS 6762 = BDS 4570 = Σ 1216	124
The orbit of ADS 7334 = JDS 1910 = A 1342	127
The orbit of ADS 7871 = BDS 5515 = O Σ 224	130
The orbit of ADS 8419 (AB) = BDS 6028 = Σ 3123	134
The orbit of ADS 8635 = JDS 2205 = A 1851	139
The orbit of ADS 9397 = A 2983	140
The orbit of ADS 12961 = JDS 3035 = A 1658	144
Appendix. Tables of $\theta_0 = \theta - \Omega$	147
Summary	187
Contents	189

Pris kr. 10:—.